The Relationship Between Radial Gas Pressure Gradient and the Planetesimal Mass

Distribution of a Protoplanetary Disk

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Abstract

The streaming instability is a mechanism that produces regions of particle overdensity in protoplanetary disks. These over-densities gravitationally collapse to form planetesimals. Although it is well known that the extent of particle clumping is dependent on the radial gas pressure gradient, the relationship between pressure gradient and planetesimal properties is not known. We carry out very high resolution local, shearing box simulations (i.e., a small co-rotating patch) of a protoplanetary disk to study the effect of the radial pressure gradient on the streaming instability and resulting planetesimal properties. We find that the pre-collapse structure of particles grows increasingly axisymmetric with increasing pressure gradient, and for relatively small radial pressure gradients, smaller filaments form with a non-axisymmetric web-like structure. The initial mass distribution can be fit to a single power-law, where we measure a power-law index of p = 1.6 for every non-zero pressure gradient. An exponentially truncated power law provides a better fit; here, we find a power-law index of p' = 1.3. We also find that the largest planetesimal masses have a weak, and possibly negligible, dependence on pressure gradient. This result rules out a cubic scaling of planetesimal mass with the pressure gradient, as suggested by linear theory. A simulation initialized with zero pressure gradient, which is not subject to the streaming instability, also yields a top-heavy mass function but with a noticeably different shape. These results point towards a initial planetesimal mass distribution that is at best very weakly dependent on the properties of the disk.

Chapter 1

Introduction

Over the past two decades, technological developments in observation and computation have contributed immensely to the knowledge of planetary formation processes. High resolution images and numerical simulations of protoplanetary disks offer an opportunity to directly study various stages of planetary formation. A common problem in planet formation theory is a variety of "barriers" that prevent the growth of dust particles into planetesimals. The bouncing barrier and fragmentation barrier are two primary obstacles that inhibit mass growth past the mm-cm size scale (Blum 2018). The bouncing barrier describes how mm-cm sized objects will bounce off each other when they collide at low enough speeds. Additionally, the fragmentation barrier describes how cm-m sized objects will break apart when they collide at speeds greater than approximately 1 m/s (Blum, 2018). Theorists work to construct explanations for why there is a large populations of meter and kilometer sized asteroids in the solar system.

In recent decades, a physical mechanism called the streaming instability has provided planetary formation theorists with a way to overcome these barriers. The streaming instability adds to planetesimal formation models by factoring in the dynamics between gas and dust in a protoplanetary disk (Youdin, 2005). Gas orbits a protostar with a sub-Keplerian velocity due to an outward, radial pressure force which counters the inward, radial gravitational force. The outward, radial pressure force is proportional to the radial gas pressure gradient (Armitage 2017). Solid objects orbit a protostar with a Keplerian velocity, which results in these objects experiencing drag from the slowly orbiting gas. This drag causes dust particles to lose angular momentum and drift radially inward. This inward drift concentrates dust particles into azimuthal regions of overdensity. Within these structures of overdensity, the roche limit is reached and direct gravitational collapse is able to form m-km size planetesimals.



Figure 1.1: This diagram of a protoplanetary disk around a protostar helps visually describe the physics that govern the streaming instability. We imagine a two-object momentum exchange between a gas particle and a dust particle. Both particles are orbiting around the protostar at the center of the diagram, however, due to the radial gas pressure gradient, the gas particle is orbiting slower than the dust particle. This velocity difference causes the dust particle to experience a drag force, and drift inward due to a loss of angular momentum.

The streaming instability is only studied theoretically and computationally. Currently,

protoplanetary disk simulations produce planetesimals with initial mass distributions that follow a power law of $dN/dM_p \propto M_p^{-p}$ where M_p is planetesimal mass, dN/dM_p is the frequency that planetesimals of mass near M_p are formed, and p is measured to be \cong 1.6 (Simon 2016). The power law index, p, is consistently calculated to be approximately 1.6 through many parameters like resolution, gas to particle ratio, and more (Simon 2017).

However, planetesimal mass distribution and range have not been thoroughly explored in relation to the strength of the streaming instability. A recent paper suggested that $M_p \propto (radial-gas-pressure-gradient)^3 (Taki, et. al. 2016)$. This suggests that the strength of the streaming instability has a direct relationship with how mass is distributed into planetesimals. This claim was made through a linear analysis of the particle surface density. However, there is no other observational or computational evidence for this claim.

This paper numerically tests the validity of this claim to explore the general relationship of a protoplanetary system's initial mass distribution with its gas pressure gradient. We run multiple numerical simulations of small regions in a protoplanetary disk with a range of pressure gradient values and all other parameters fixed. This paper shows that there is little to no relation between gas pressure gradient and planetesimal mass range. We also find that when fit to a single power law function, the mass distribution has a power-law index of $p \approx 1.6$. However, when looking at the cumulative distribution, we find that a better fit to the distribution is a truncated power-law function with a new index of: $p' \approx 1.3$. We also make qualitative observations about the particle density structure of the disk under different pressure gradients.

Chapter 2

Methods

To simulate the formation of planetesimals in a protoplanetary disk we use the magnetohydrodynamical (MHD) Athena code to couple the motion of gas and dust in a small region of the disk. We model this space of the disk to be a co-rotating shearing cube. We also approximate this cube to be small relative to the the size of a protoplanetary disk, and exist far from the center of the disk. These approximations allow us to model the box in cartesian coordinates rather than spherical. More specifically, we interpret the radial coordinate to x, the azimuthal coordinate t y, and the altitude to z. The x-axis has periodic-shear boundary conditions, the y-axis has periodic boundary conditions, and the z-axis has outflow boundary conditions.



Figure 2.1: This diagram displays what our co-rotating shearing box actually simulates with respect to a global simulation of a protoplanetary disk. The term "co-rotating" refers to how the box that we simulate orbits with respect to the protoplanetary disk. By simulating just a small portion of a protoplanetary disk, we are able to compute much higher resolution systems.

Within this modified coordinate system, Athena solves the continuity (1) and momentum (2) equations for gas dynamics:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0, \tag{1}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \boldsymbol{u} + P \boldsymbol{I}) = 2q_{\rm sh} \rho \Omega^2 \boldsymbol{x} - \rho \Omega^2 \boldsymbol{z} -2\boldsymbol{\Omega} \times \rho \boldsymbol{u} + \rho_p \frac{\boldsymbol{v} - \boldsymbol{u}}{t_{\rm stop}}$$
(2)

where ρ is gas density, ρu is momentum density, *P* is gas pressure. $q_{sh} = 3/2$ and is the shear parameter. ρ_p is the mass density of particles, *v* is particle velocity, and t_{stop} is the stopping time parameter that influences the speed a particle will lose momentum. Also, pressure, *P*, can be expressed as ρc_s^3 where c_s is isothermal sound speed. The final governing equation is the equation of motion for a given particle, *i*:

$$\frac{d\boldsymbol{v}_{i}'}{dt} = 2\left(v_{iy}' - \eta v_{\rm K}\right) \,\Omega \hat{\boldsymbol{x}} - (2 - q) \,v_{ix}' \Omega \hat{\boldsymbol{y}} \\ -\Omega^{2} z \hat{\boldsymbol{z}} - \frac{\boldsymbol{v}_{i}' - \boldsymbol{u}'}{t_{\rm stop}} + \boldsymbol{F}_{\rm g}$$
(3)

Where $\eta v_{\rm K}$ accounts for inward radial drift of particles. This inward radial drift is caused from the radial gas pressure gradient. We will represent radial gas pressure gradient with the dimensionless value $\Pi = \eta v_{\rm K}/c_{\rm s}$.

All of the simulations used in this paper were run on the *Stampede2* supercomputer at the University of Texas, Austin. Each simulation also ran with the following parameters: Z = 0.1, $\tau =$

0.05, and G = 0.02. Z is the ratio between particle and gas mass, τ is the dimensionless stopping time of particles through gas, and G is the strength of gravitational tidal forces between particles. G is not purely changing the gravitational force, but is meant to model particle self gravity with respect to inward radial gravitational force from the protostar. We ran six simulations with varying pressure gradients values and particle self-gravity on the entire simulation time. In the Athena code, pressure gradient is modeled purely by applying an inward radial velocity to particles in the simulation. The six Π values that we used were: 0.0, 0.0375, 0.05, 0.0625, 0.075, 0.0875, and 0.1. Also, the $\Pi = 0.1$ case became so turbulent that we needed to increase the height of the z boundaries so mass would not escape the simulation. We ran three additional simulations with particle self-gravity turned on after the streaming instability had fully created overdense structures. For these late particle self-gravity simulations, we used $\Pi = 0.0375, 0.05, 0.05, 0.075$.

Run	Па	${t_{\rm sg}}^{b}_{(\Omega^{-1})}$	p^{c}	p'^{d}	$M_0/M_G^{\rm e}$	$M_{50}/M_G{}^{ m f}$	Mass Fraction in Planetesimals
P0-SG0	0	0	1.75 ± 0.03	1.52 ± 0.05	0.78 ± 0.21	0.57	0.81
P0.0375-SG0	0.0375	0	1.64 ± 0.02	1.28 ± 0.03	0.26 ± 0.02	0.31	0.66
P0.05-SG0	0.05	0	1.60 ± 0.03	1.29 ± 0.03	0.35 ± 0.05	0.42	0.55
P0.0625-SG0	0.0625	0	1.59 ± 0.03	1.31 ± 0.04	0.32 ± 0.04	0.35	0.29
P0.075-SG0	0.075	0	1.58 ± 0.05	1.31 ± 0.06	0.56 ± 0.17	0.61	0.22
P0.0875-SG0	0.0875	0	1.58 ± 0.07	1.33 ± 0.09	0.82 ± 0.34	1.0	0.12
P0.1-SG0-Lz0.4H	0.1	0	1.49 ± 0.07	1.21 ± 0.07	0.44 ± 0.12	0.52	0.073
P0.0375-SG72	0.0375	72	1.56 ± 0.02	1.19 ± 0.03	0.16 ± 0.02	0.22	0.53
P0.05-SG80	0.05	80	1.64 ± 0.03	1.30 ± 0.05	0.24 ± 0.04	0.28	0.28
P0.075-SG66	0.075	66	1.51 ± 0.06	1.21 ± 0.06	0.26 ± 0.05	0.39	0.062

^a Pressure gradient parameter, defined by Equation 7

^b Time (in units of Ω^{-1}) at which particle self-gravity is initiated

^c Best fit power law slope to single power law model

^d Best fit power law slope to truncated power law model

^e Best fit truncation mass in truncated power law model

^f The mass at which $M_{p,tot,n}(>M_p) = 0.5$, where $M_{p,tot,n}(>M_p)$ is the total planetesimal mass greater than M_p normalized to the total mass in planetesimals.

Figure 2.2: This table displays where in parameter space we ran all of our simulations, as well as the numerical results that we calculated from them. Under the "Run" column the first letter "P" refers to Π and the "SG" refers to start gravity time. The one time where "Lz" is placed refers to the length of the shearing box along the z-axis.

For all of these simulations we qualitatively chose a time to run a mass finding algorithm

by looking at the particle structures in each system to determine whether planetesimals had fully

collapsed. The time at which we chose to extract our planetesimal data is called the fiducial time. For the six simulations that have gravity turned on from the start, we also have data points taken at certain times before the fiducial time (as seen in Sec. 3.1). Also, our mass finding algorithm, PLanetesimal ANalyzer (PLAN), creates a list of masses, Hill sphere's, and x/y/z coordinates for each planetesimal identified from the particle information files. We convert our planetesimal mass values to units of M_G , the gravitational mass to remain consistent. To normalize as such, we use the standard Toomre dispersion relation by balancing tidal and self-gravitational forces to obtain a critical unstable wavelength of

$$\lambda_G = \frac{4\pi^2 G \Sigma_p}{\Omega^2} \,. \tag{4}$$

Gravity will overpower tidal shear for planetesimals with a diameter smaller than λ_G . Thus, we can define a gravitational mass as a patch of the particle disk with surface density Σ_p and diameter λ_G .

$$M_{G} \equiv \pi(\frac{\lambda_{G}}{2})^{2} \Sigma_{p} = 4\pi^{5} \frac{G^{2} \Sigma_{p}^{3}}{\Omega^{4}} = \frac{\sqrt{2}}{2} \pi^{9/2} Z^{3} G^{2} M_{H}, \qquad (5)$$

where $M_H = \rho_0 H^3$ is a dimensional reference mass.

Chapter 3

Results

3.1 Pressure Gradient Influence on Particle Structure

By looking at the time evolution of different pressure gradient runs, we were able to make qualitative observations about the structure of particles in the shearing box. We found that lower pressure gradient runs produced asymmetric, web-like precollapse structure of the particles. On the other hand, higher pressure gradient values result in axisymmetric precollapse structure. This formation of particles in the disk is strongly associated with the streaming instability. The elongated particle structure in the high pressure gradient runs is exemplary of the regions of overdensity that induce planetesimal formation. As seen in figure 3.1, most of the planetesimals forming in the high pressure gradient runs lie in the elongated regions of high particle density.



Figure 3.1: Surface density plots for three of the pressure gradient values that we simulated. Each row is a different pressure gradient simulation with the Π value to the right. This figure also shows the time evolution of each simulation in the top right hand corner of each snapshot. Recall that Ω^{-1} refers to the time of one orbit. Thus, each run in this figure evolves from left to right over time.

3.2 The Direct Relation Between Maximum Planetesimal Mass and Radial Pressure Gradient

These results for maximum planetesimal size (M_p) were the first results that opposed the claim that maximum M_p scales with Π^3 . In Figure 3.2, we overplot a dotted line representing Π^3 to show the trend that we would expect. However, we clearly see a relatively scattered distribution of maximum M_p values with relation to Π . Another interesting observation is the trend between time and maximum M_p . For all applicable Π values, the M_p value for fiducial time - 10 Ω^{-1} is lower than the M_p value for fiducial time - 5 Ω^{-1} . Both of the earlier values are also always lower than the maximum M_p value at the fiducial time. Although this initial observation seems to provide substantial evidence against the Taki claim, there are very few maximum planetesimal mass values per Π value. And given the chaotic nature of these systems, we cannot fully trust these points without error bars. We will produce a statistically stronger measure for a system's most massive planetesimals in Sec. 3.4.



Figure 3.2: Maximum planetesimal mass vs. Π . The black circles, blue triangles, and red asterisks all represent simulations with particle self-gravity on from the start, while the green squares represent simulations with particle self-gravity turned on later. Recall that the inverse omega symbol represents the unitless time value that the simulation uses. The blue triangle and red asterisk are data points taken a time earlier than the fiducial time.

3.3 The Differential Mass Function and Single Power Law Indices

The differential mass function of initial planetesimals formed is a valuable quantitative measure. In the differential mass function, each data point is a planetesimal that has been identified by PLAN. Figure 3.3 displays the maximum planetesimal mass as the rightmost point in each simulation. The colored lines also represent the power-law fit for each mass distribution. Each fit function is simply $C^*(M_p^{-p})$ where -p represents the slope of the line in log-log space, and C represents an arbitrary constant that changes the height of the function.



Figure 3.3: Differential mass distribution taken from three different pressure gradient simulations run with particle self-gravity on from the start. The three Pi values are: 0.0375 (blue pluses), 0.075 (black asterisks), and 0.1 (red squares).

The p values were found using the maximum likelihood estimate. We find that the power law index is approximately 1.6 over a wide range of Π values as seen in figure 3.3.



Figure 3.4: Power-law index values (p) plotted with respect to the Pi values from each simulation. In this specific plot, the values are only from SG0 simulations. A dotted line is also overplotted at p=1.6.

Every SG0 simulation except for Pi=0.0 lies within 2 sigma of p=1.6. The streaming instability does not occur for a system with no pressure gradient, so this is not much of a concern. Also, a reason for such a low p value for the Pi=0.1 run might be the significantly smaller number of planetesimals produced in that system.

3.4 The Cumulative Mass Function and Exponentially

Truncated Power-Law Fit

When plotting the cumulative mass function of our simulation data we find that a single power-law is not the best descriptor of the mass distribution. We find that an exponentially truncated mass function of the form $N(>M_p) \propto M_p^{p'-1} \exp[-M_p/M_0]$ best fit the cumulative data.



Figure 3.5: Cumulative mass function of simulations fit to: N(>M_p) $\propto M_p^{p'-1} \exp[-M_p/M_0]$

This new fit introduces two new parameters, p' and M_0 . p' represents the power-law slope for the low mass regime of planetesimals, while M_0 is the characteristic mass value that represents the exponential drop off at the high mass regime of planetesimals.

We use M_0 as an analog to the largest planetesimals produced in a simulation. M_0 is a more powerful measure of largest characteristic mass for a set of planetesimals because it takes the entire regime of largest planetesimals into account rather than only one.



Fig. 3.6: M_0/M_G vs. Π with a constant, linear, and cubic function overplotted.

Even when using M_0 as a measure of characteristic mass, the value does not increase proportional to Π^3 . Overall, we find M_0 to have a weak linear dependence on Π . There is even a possibility that the Π parameter plays no role in determining this characteristic of the planetesimal distribution.

Chapter 4

Conclusion

Our initial observations of the particle structure of shearing box simulations with different pressure gradients supported our understanding of the streaming instability. Specifically we found that precollapse structure was asymmetric for low pressure gradient runs, but produced an azimuthal shape for high pressure gradient runs. We find that when planetesimal formation simulations with varying radial gas pressure gradients are fit to a single power-law, $dN/dMp = Mp^{\phi}$, $p \approx 1.6$. We also find that the mass function is better fit to an exponentially truncated power-law, $N(>M_p) \propto M_p^{p'-1} \exp[-M_p/M_0]$, where we find a low mass power-law index of p' = 1.3 and a characteristic planetesimal mass, M_0 , that has a weak, and possibly negligible, dependance on pressure gradient. Along with direct mass measurements, we rule out the concept that planetesimal mass is proportional to pressure gradient cubed, which was suggested by linear theory (Taki, et. al. 2016).

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