

Elastic Interactions between Optical and Topological Solitons in Chiral Nematic Liquid Crystal

Andrew J. Seracuse*

Honors Undergraduate Thesis

Thesis Advisor: Ivan Smalyukh (Physics)

Honors Council Committee Representative: Michael Ritzwoller (Physics)

Out-of-Department Committee Representative: Mark Keller (NIST)

Unofficial Committee Representative and Graduate Student Advisor: Andrew Hess (Physics)

Defense Date: 10 April 2017

* University of Colorado Boulder, Department of Physics, 2000 Colorado Ave., Boulder, CO 80309; Email: Andrew.Seracuse@Colorado.edu

Abstract

The research presented in this thesis investigates the elastic director field interactions between optical, spatial solitons and defect structures in frustrated chiral nematic liquid crystals. I characterize experimentally the rich elastic interactions between *torons*, particle-like localized director field configurations with non-trivial topology, and spatial, optical solitons, which have been previously dubbed *nematicons*. By exploitation of the waveguide regime for the generation of stable thresholdless nematicons via the reorientational nonlinearity present in thermotropic liquid crystals, I characterize the elastic potential energy surrounding a stationary nematicon and a mobile toron via measurement of toron displacement as a function of time. I compare the resultant elastic potential energy contour map with direct optical characterization of the integrated elastic director perturbations within the nematicon. With knowledge of the underlying nematicon structure, I show that nematicon beam deflection from spatially fixed torons is possible.

Table of Contents

1. Introduction.....	1
1.1 Liquid Crystal as a Phase of Matter.....	2
1.2 Frustrated Chiral Nematic Liquid Crystal and Topological Solitons	6
1.3 Nematicons	8
2. Experimental Methods.....	10
2.1 Materials' Preparation	11
2.2 Generation of Topological Solitons with Laser Tweezers	13
2.3 Experimental Observation of Nematicons and Topological Solitons with Polarizing Optical Microscopy	15
2.4 Data Analysis	18
3. Experimental Results.....	24
3.1 Generation of Topological Solitons	25
3.2 Generation and Analysis of Nematicons	26
3.3 Elastic Interactions between Stationary Nemations and Mobile Torons	27
3.4 Nematicon Deflection with Stationary Topological Solitons	29
4. Conclusion	30
5. References	33

1. Introduction

Liquid crystals are a ubiquitous constituent of the information age. From smart phone screens to supercapacitors [1], the ordering caused by their proximate attractive interactions and repulsive steric ones among geometrically anisotropic particles extends to the meso- and even macro-scale to enable modern technological advancement. Of particular interest for this thesis is the characterization of novel liquid crystal systems and their enabling physics which allow the facile configuration of and control over all-optical interconnections and light-induced waveguides. While such systems have been realized with self-focusing optical beams called spatial, optical solitons (*nematicons* in nematic liquid crystals) and their control with external or embedded perturbations, these prior systems focus on the control of nematicons with finite dielectric perturbations [2], external voltages [3], external laser beams [4], interactions with other nematicons [5], interactions with interfaces [6] and cell boundaries [7], and self-acceleration [8].

I demonstrate control over nematicon propagation with elastic interactions between nematicons and topological solitons. I show that topological solitons may be written in arbitrary arrays in real time to cause nematicon deflection with unprecedented control, the potential for system miniaturization, and pre-engineered system self-configuration. First, I discuss liquid crystals in an introductory context (§1.1) with specialization to frustrated chiral nematic systems with topological solitons (§1.2). Next, I describe nematicons (§1.3) with emphasis placed on my experimental system. In §2, I discuss the experimental methods that I employed to conduct my research. Specifically, I describe materials' preparation (§2.1), the generation of topological solitons with laser tweezers (§2.2), experimental observation of nematicons and topological solitons with optical microscopy techniques (§2.3), and data analysis (§2.4). In §3, I show my experimental results which include arrays of topological solitons (§3.1), nematicons in homeotropically aligned frustrated chiral nematic liquid crystal (§3.2),

translation of particle-like topological solitons (*torons*) with a stationary nematicon (§3.3) thereby mapping the elastic potential energy of the toron near the nematicon, and deflection of nematicons from stationary torons and other topological solitons (§3.4).

1.1 Liquid Crystal as a Phase of Matter

While matter may be generally classified into the well-known five states, intermediate mesophases of matter can exist between these classical states. Liquid crystals, per their definition, exhibit this mesophase behavior as they are liquids with molecular ordering [9]. The reason for this anisotropic organization arises from the geometric anisotropies of liquid-crystal molecules. Because of van-der-Waals-like interactions in *thermotropic* liquid crystals, whose organizational liquid phase is transformed principally through temperature, and charge-screening and steric interactions in *lyotropic* liquid crystals, whose phase behavior is primarily modified through molecular concentration of liquid crystal solute in solvent host, the geometric anisotropy of the constituent molecules can cause highly non-localized, ordered liquids [10]. As a general principle, order of these mesophases increases as temperature decreases, with classical crystalline solids and isotropic liquids representing terminal phases of liquid crystals at very low and very high temperatures, respectively.

I immediately specialize to consider thermotropic, rod-shaped liquid crystals with *chiral dopants*—chiral liquid-crystal molecules which are added to a non-chiral host liquid crystal in order to induce helicoidal twisting in the mixture. As shown in Fig. 1, the *nematic* phase is one mesophase of rod-shaped liquid crystals. Nematic liquid crystals have orientational ordering only; that is, the long axes of the molecules, on average, point in the same direction but their centers of mass are randomly distributed. This average orientation can be described by the nonpolar director field $\mathbf{n} \equiv -\mathbf{n}$. One can define a scalar order parameter S which is a function of the angle between the director and the average deflection angle between the director and the long axis of the molecules on average [11]. (See Fig. 2.)

Upon reduction of temperature, the nematic phase may transition to the *smectic* phase, wherein 1D positional ordering, as well as orientational ordering, is achieved. Finally, of interest to this paper, the *cholesteric* (chiral nematic) phase may be described by two orientational degrees of freedom [12]. The nematic orientational ordering of the long axes of the liquid crystal molecules tends to twist about a chiral axis χ , which is perpendicular to the long axes of the molecules. The degree of helicoidal twisting may be described by their *pitch*—the distance over which the molecules rotate by 2π about the chiral axis.

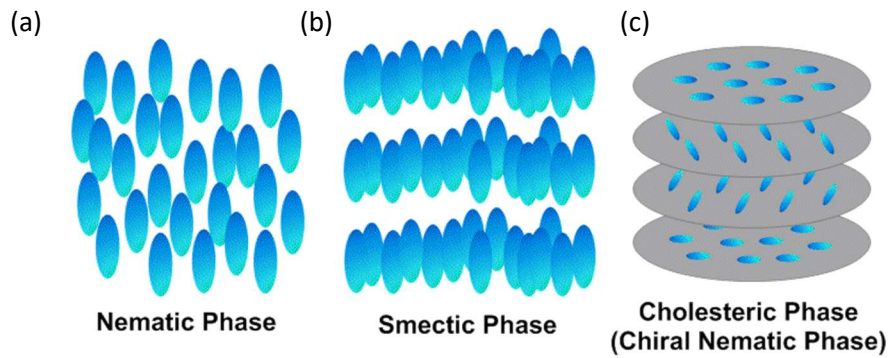


Figure 1: Mesophases of liquid crystal. (a) Nematic phase which is characterized by orientational but no positional order. (b) Smectic phase which has orientational order and 1D positional ordering. (c) Cholesteric phase which is characterized by two degrees of orientational ordering: orientational ordering as in the nematic phase and rotation about its chiral axis. Adapted from [13]

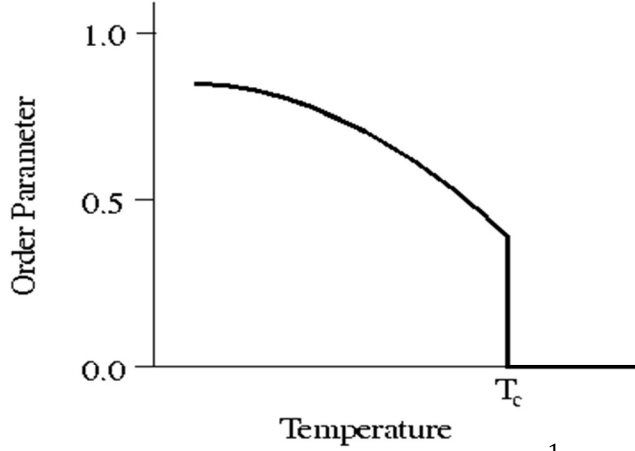


Figure 2: The scalar order parameter is defined as $S = \frac{1}{2} \langle 3 \cos^2(\theta) - 1 \rangle$ where θ is the angle between the liquid crystal director and the individual long axis of each liquid crystal molecule. The brackets indicate the average molecular value of orientation. When the molecules are colder there will be fewer thermal fluctuations from the director, which will lead to a higher order parameter. The order parameter will gradually decrease as temperature is increased until the critical temperature is reached at which all order disappears and the liquid crystal transitions from the nematic to the isotropic phase. Adapted from [11].

The director field of liquid crystal is highly nonlocal with orientational and, perhaps, positional ordering extending in spatial scales several orders of magnitude larger than the dimensions of the individual molecules. Because of the elastic attraction between neighboring molecules, there is an energetic cost for perturbations of the director. Therefore, when nematic liquid crystal is placed between two boundaries with well-defined director conditions, it is energetically favorable for the liquid crystal in that region to align with those boundary conditions and maintain the director defined at the boundaries throughout the bulk of the liquid crystal. One may quantify the energetic cost of various director perturbations in a uniaxial nematic liquid crystal system through Euler-Lagrange minimization of the Frank-Oseen elastic free energy which is defined as $F = \frac{1}{2} \int_V dV \left\{ K_{11} (\nabla \cdot \mathbf{n})^2 + K_{22} (\mathbf{n} \cdot (\nabla \times \mathbf{n}))^2 + K_{33} (\mathbf{n} \times (\nabla \times \mathbf{n}))^2 - K_{24} \nabla \cdot (\mathbf{n} \cdot (\nabla \cdot \mathbf{n}) + \mathbf{n} \cdot (\nabla \times \mathbf{n})) \right\} - \frac{1}{2} \int_V dV \left\{ \epsilon_0 \Delta \epsilon (\mathbf{E} \cdot \mathbf{n})^2 + \frac{\Delta \chi}{\chi_0} (\mathbf{B} \cdot \mathbf{n})^2 \right\} + \frac{1}{2} W_0 \int_S dS \{ S^2 (\theta - \theta_0) \}$, where the first volume integral on the right accounts for free energy from elastic perturbations of the director field, the second volume integral describes the free energy from

electric ($\mathbf{E}$) or magnetic field ($\mathbf{B}$) interactions (if present) with the liquid crystal, and the final surface integral describes the free energy conditions imposed by angular alignment of the liquid crystal with boundary conditions (θ) [14]. The various Frank elastic constants K_{ii} account for the energy differences required to form different elastic perturbations, namely, splay, twist, bend, as shown in Fig. 3, and saddle splay (not shown) while the anchoring energy W_0 describes the difficulty of perturbing the equilibrium boundary-condition director.



Figure 3: Common types of elastic director perturbations including splay, twist, and bend as indicated in the image.

Another important feature of geometrically anisotropic liquid crystal molecules is their birefringence which results from differing electric susceptibilities along different directions of liquid-crystal molecules [11]. For rod-like molecules, incident light experiences an ordinary index n_o when the polarization of the light is perpendicular to the molecules' long axes. Additionally, when the polarization of incident light is parallel to their long axes, an extraordinary index n_e may be defined. If the angle θ between the molecular long axis and the polarization of the light is between parallel and perpendicular, then the effective index of refraction will be governed by the equation $n_{eff}^2 = \frac{n_e^2 n_o^2}{n_e^2 \cos^2 \theta + n_o^2 \sin^2 \theta}$ [11].

In the next section, I show how considerations of director alignment and elastic energy costs of director perturbations play a crucial role in the enabling properties of topological solitons in my liquid crystal system. Subsequently, I explain the importance of birefringence for the generation of nematicons in liquid crystals.

1.2 Frustrated Chiral Nematic Liquid Crystal and Topological Solitons

When chiral nematic liquid crystal is subjected to boundary conditions, the resulting bulk director structure is caused by the energetic balancing between alignment with the boundary conditions and alignment with the intrinsic helicoidal twist tendency within a chiral nematic liquid crystal. A *frustrated chiral nematic liquid crystal* exists when boundary conditions prohibit twist along the thickness between the boundaries d , as is the case of homeotropic alignment of the boundary conditions [14]. (See Fig. 4(a) below.)

The equilibrium configurations of frustrated chiral nematic systems may be predicted through the minimization of the Frank-Oseen free energy for chiral nematic liquid crystal $F = \frac{1}{2} \int_V dV \left\{ K_{11} (\nabla \cdot \mathbf{n})^2 + K_{22} \left(\mathbf{n} \cdot (\nabla \times \mathbf{n}) + \frac{2\pi}{p} \right)^2 + K_{33} (\mathbf{n} \times (\nabla \times \mathbf{n}))^2 - K_{24} \nabla \cdot (\mathbf{n} \cdot (\nabla \cdot \mathbf{n}) + \mathbf{n} \cdot (\nabla \times \mathbf{n})) \right\} - \frac{1}{2} \int_V dV \left\{ \epsilon_0 \Delta \epsilon (\mathbf{E} \cdot \mathbf{n})^2 + \frac{\Delta \chi}{\chi_0} (\mathbf{B} \cdot \mathbf{n})^2 \right\} + \frac{1}{2} W_0 \int_S dS \{ \sin^2(\theta - \theta_0) \}$ where we note that the twist (K_{22}) term has been changed from $K_{22} (\mathbf{n} \cdot (\nabla \times \mathbf{n}))^2$ to $K_{22} \left(\mathbf{n} \cdot (\nabla \times \mathbf{n}) + \frac{2\pi}{p} \right)^2$ in order to account for the intrinsic helicoidal twist tendency in chiral nematic liquid crystal [14]. By minimization of the free energy in the absence of external fields and with $0 < \frac{d}{p} \leq 1$, it has been found that topologically complex field configurations will form within certain $\frac{d}{p}$ regimes in order to form an equilibrium director ground energy state [15]. (See Fig. 4(b).) For example, if $\frac{d}{p} < \sim 0.8$, then the minimum energy configuration is untwisted chiral nematic liquid crystal, *i.e.* the homeotropic nematic state wherein the liquid crystal molecules align with the homeotropic boundary conditions despite the presence of the chiral dopant. If $\sim 0.8 \leq \frac{d}{p} < \sim 1$, particle-like localized director-field configurations form with bend, splay, and saddle splay terms that are less energetically costly than twist deformations are. Finally, if $\frac{d}{p} \sim 1$, localized, translationally invariant director-field configurations with linear extent will form with maximized

contributions from bend and splay terms to minimize the energetically expensive twist term. The latter two cases are examples of topological solitons in liquid crystals. Particle-like localized field configurations are referred to as *torons* because of their double-twist toroidal cylinder director configuration while localized field configurations with linear extent are called *cholesteric fingers* [16].

While topological solitons may be free energy ground states for certain thickness-to-pitch regimes, a significant energy barrier much larger than $k_B T$ often exists between the untwisted and solitonic states. In order to generate topological solitons, a stimulus which perturbs the director must be provided, as indicated in Fig. 4(c). In my research, I considered two possible methods to induce this distortion. The first was to heat the sample so that the energy barrier is $\sim k_B T$. When the sample's temperature is increased, the order parameter decreases. When the order parameter is low enough, the induced distortion becomes large enough to reorder the system to the ground-state director structure. The other possible method is to apply an electric field to the liquid crystal. This can be done either by applying a voltage across liquid crystal with capacitive plates or by inserting a light beam into the liquid crystal where the electric field direction is determined by the polarization of the wave. This induces a reorientation by induced polarization of liquid crystal molecules and subsequent electric torque.

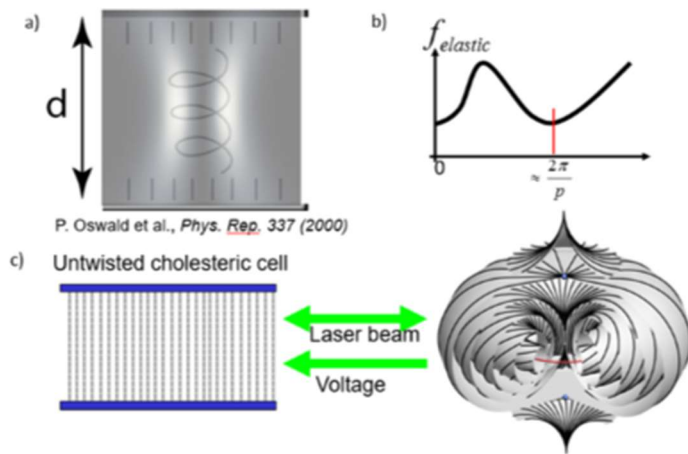


Figure 4: (a) Depiction of frustrated chiral nematic liquid crystal. The intrinsic helicoidal twist competes with the homeotropic boundary conditions during the formation of various field configurations. (b) A plot of elastic free energy density as a function of twist. For $p \rightarrow \infty$ (untwisted) and $p \sim d$, energy minima exist. (c) A localized stimulus, such as a laser beam, perturbs the liquid crystal through heating and electric torque to form topological solitons such as a toron.

In the next subsection, I will discuss the optical counterpart to topological solitons and provide background understanding, in addition to that provided in this subsection, for the elastic interactions between topological solitons and nematicons.

1.3 Nematicons

A quasi-stable, localized director-field perturbation analogous to topological solitons in an unwound chiral nematic liquid-crystal far field is the structure known as a spatial, optical soliton or *nematicon*. As is the case for all optical solitons regardless of the nonlinear self-focusing mechanism or transverse dimensionality, their existence hinges on the exact balancing of self-focusing and diffraction within the medium through which they propagate [17]. If not for the effects of the self-focusing reorientational nonlinearity in liquid crystal, the beam would diffract as it propagates through its host medium as shown in Fig. 5(a) [16]. However, because a higher index of refraction, proportional to optical intensity, is experienced by the beam as liquid-crystal molecules are rotated in response to the torque from the light's electric field, a lensing effect which causes self-focusing occurs as shown in Fig. 5(b).

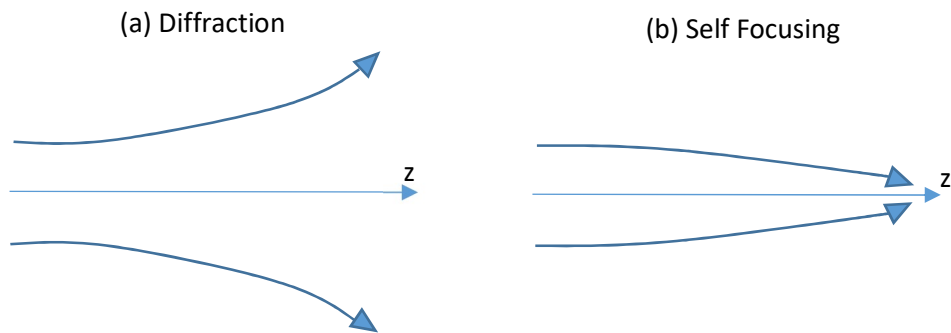


Figure 5: The effects of diffraction (a) and self-focusing (b) in liquid crystal cells. The curving arrows above and below the z axis are the edge of the beam.

One may gain insight into the behavior of nematicons by the consideration of the equations of motion for nematicons in homeotropically aligned liquid-crystal systems. From Karpierz [18], I assume the experimental configuration as shown in Fig. 6. For a linearly polarized incident beam, the pure TM mode causes no reorientation of the liquid crystal and no solitonic beam. With a pure TE mode, which is the polarization condition for causing a perturbation of the liquid crystal only beyond the Freedericksz threshold [14], it is reported that such nematicons are unstable with “undulation, filamentation, and break-up of the light beam” [18] in homeotropically aligned nematic liquid crystal [19-21].

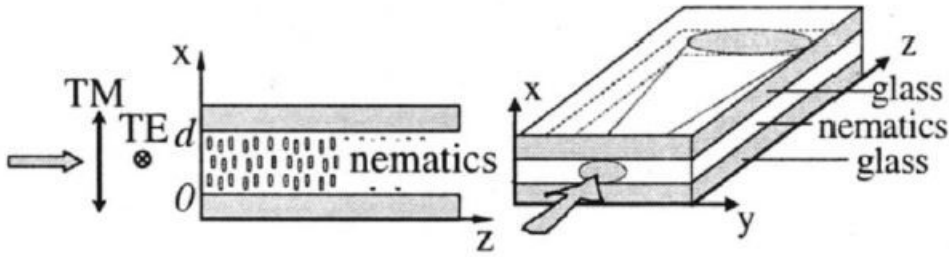


Figure 6: The experimental configuration for launching nematicons in homeotropically aligned liquid crystal. Adapted from [18].

Interestingly, experimentally observed stable nematicons in the literature propagating in the geometry of Fig. 6 with sufficiently small d were the thresholdless type whose incident polarization was a superposition of TM and TE modes [18, 22-24]. As a starting point for theoretical analysis, one may consider the perturbative regime wherein weak reorientation of the liquid crystal director occurs to form a nematicon with no threshold intensity. The equations of motion found in [18] and [22] incorporate TM and TE modes of the confining glass waveguide used to define the homeotropic boundary conditions for the nematic liquid crystal. Even with neglecting the surface energy contribution from the confining homeotropically aligned substrates, the correct distribution of undulating light intensities among the modes along their propagation axis is predicted. For homeotropic waveguided

nematicons, the simple relationship that describes the undulation in intensity and director configuration along the propagation axis is $L_B = \frac{\lambda}{N_e - N_o} \geq \frac{\lambda}{n_e - n_o}$, where L_B beat length of the periodic undulation, λ is the wavelength of the CW laser used to form the nematicon, N_i are the effective extraordinary and ordinary indices of refraction experienced by the nematicon, and n_i are the extraordinary and ordinary indices of refraction for the liquid crystal [22].

For the purposes of this paper, an important consequence of the director perturbation caused by the self-focusing effect of the nematicon is that the perturbation of the director does not immediately disappear at the edge of the beam but has a rather long transverse decay length. This is because elastic deformations of the director structure are a spatially nonlocal effect. Hence, the elastic interactions between nematicons and topological solitons are long-range and extend beyond the visible birefringence present in nematicons and topological solitons under polarizing optical microscopy.

In the next section, I will discuss the experimental methods that I used to prepare materials, to generate topological solitons with laser tweezers, to observe topological solitons and nematicons with optical microscopy techniques, and to perform data analysis techniques which enabled the mapping of the toron-nematicon interaction potential energy and observation of the deflection of nematicons from topological solitons.

2. Experimental Methods

In this section, I will discuss the way that the experiment was carried out. First, I will talk about the procedure used to prepare the samples. Next, I will discuss the way that I induced topological solitons in the sample and the equipment used to do this. I will then cover the equipment and methods used to observe and record video of the interaction. Finally, I will explain the video-analysis techniques

and the mathematical methods used to evaluate the forces at play in the elastic interaction between the nematicon and the toron.

2.1 Materials' Preparation

In order to observe elastic interactions between nematicons and topological solitons, I needed to construct a liquid-crystal *cell*—confining pieces of glass with defined boundary conditions and separation for a known thickness—that was amenable to both solitonic structures. To make the cell that was used for the interaction of a stationary nematicon and mobile torons, I cut pre-cleaned 1-mm-thick microscope slide glass into 25 mm x 12.5 mm rectangles and polished the long manufacturer's edges as shown in Fig. 7 with a 9- μ m coarseness polishing disk on a lapping machine (Buehler Ecomet 3) at 100 rpm. (This was to ensure uniform liquid-crystal defect formation along the nematicon's optical interface to guarantee a repeatable nematicon.) Next, the glass was treated with 1 wt. % aqueous dimethyloctadecyl[3-(trimethoxysilyl)propyl] ammonium chloride (DMOAP, Acros Organics) and washed with deionized water. As shown in Fig. 8, the first piece of glass was positioned on the curing stage with the polished edge facing up and toward the back of the curing stage. UV-curable optical adhesive (Norland Optical Adhesive 65, Norland Products, Inc.) was mixed with $\sim$ 15.9- μ m-thick silica spacers (Thermo Scientific) and placed as shown in Fig. 7. Subsequently, the top piece of glass was placed over the UV-glue dots with its polished edge facing down and toward the back of the curing stage. Both pieces of glass were gently pushed toward the back of the curing stage. I then waited for 30 minutes to allow the spacers to settle in order to obtain the most consistent distance between the two pieces of glass. Finally, I cured the cell by exposing it to UV light for about 30 minutes.

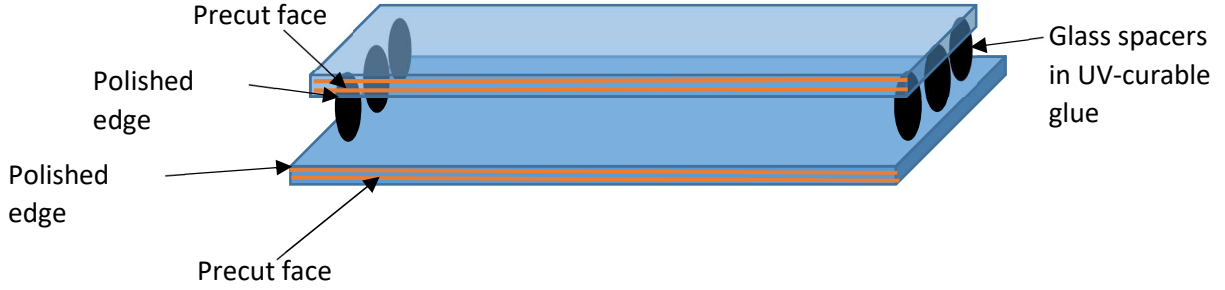


Figure 7: Liquid crystal cell before being filled with liquid crystal.

Alternatively, ~200- μm -thick indium-tin-oxide (ITO) coverslips were cut in half so that the pieces measured 22 mm x 11 mm. Pre-cleaned coverslips and 25 mm x 12.5 mm slide glass were polished as described above. Both were treated with SE-1211 (Alpha Micron, Inc.) via spin coating (ramp to 700 rpm and hold for 1 s, ramp to 3000 rpm and hold for 30 s) and cured in an oven which was ramped to 185 °C over 30 min. and held at 185 °C for 1 hr. The slide glass was placed first in the curing stage, covered with UV-glue dots with silica spacers as shown in Fig. 7, and finished by placing the coverslip ITO-side down with its polished edge also down and toward the back of the curing stage. The cell was cured in the same manner as described above.

After the glue was cured, I measured the distance between the two pieces of glass by optical interference. While the spherical silica spacers do provide a nominal separation of their average diameter, I needed a precise measurement of the thickness in order to formulate chiral nematic liquid crystal with an appropriate pitch. I used cholesterol pelargonate (Sigma Aldrich) as the chiral dopant with a helical twisting power (HTP) of 6.25 μm^{-1} for E7 liquid crystal (Shijiazhuang Chengzi Yonghua Display Material Co., Ltd.) and formulated the final pitch of the chiral nematic liquid crystal according to the relationship $c = \frac{1}{HTP \cdot p}$, where c is the concentration of the chiral dopant and p is the desired pitch in μm . As the dopant's host, I used E7 because of its high nematic-to-isotropic melting temperature (61.5 °C) to ensure the robust generation of nematicons under focused laser light. For the generation of

torons, I used a thickness-to-pitch ratio of ~ 0.9 whereas for the generation of cholesteric fingers, I used a ratio of ~ 0.98 to 1.

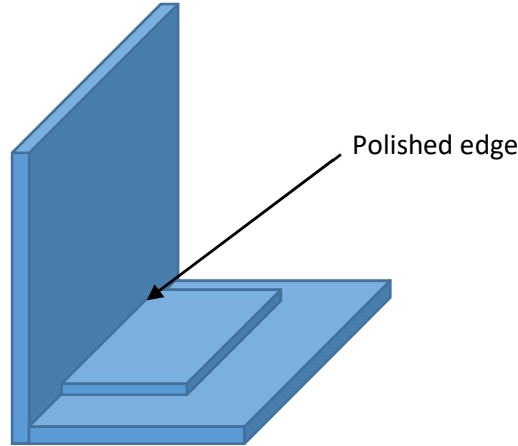


Figure 8: Position of bottom piece of glass in curing stage.

2.2 Generation of Topological Solitons with Laser Tweezers

As discussed in §1.2, torons may be generated when a sufficient stimulus exists to overcome the energy barrier between the untwisted and solitonic states [24]. While the untwisted chiral nematic liquid crystal may be heated past the nematic-to-isotropic transition temperature and rapidly quenched back to room temperature, this method does not allow one to control the spatial positions of the resultant topological solitons. Also, because E7 has a positive dielectric anisotropy, the application of an electric field across capacitive ITO-glass plates through the thickness of the cell would cause no change in director alignment.

To write precise arrays of torons and cholesteric fingers in frustrated chiral nematic liquid crystal, I used laser tweezers to perturb the liquid crystal director in a spatially controlled manner. (See Fig. 9.) Because the 1064-nm laser suffers absorption by the liquid crystal and the ITO substrate, both thermal fluctuations and electric torque reoriented the liquid crystal at the laser's focal point within the

cell thickness. With a proper thickness-to-pitch ratio, distorting the director structure can induce a ground-state topological soliton at arbitrary lateral positions within the cell.

For observation of the interaction between stationary nematicons and mobile torons, slide glass with DMOAP was used to construct the cell. The sample cell was placed on a heating stage and heated to ~ 50 °C during operation of the laser tweezers. With a 10x objective (see Fig. 9), the laser was focused into the liquid crystal with about 2 W of power to sufficiently distort the director to form torons in a cell with a thickness-to-pitch ratio of ~ 0.9 . In a cell with a ratio of about ~ 1 , cholesteric fingers were drawn with the same amount of power.

In order to study the deflection of nematicons from fixed torons, one piece of the cell glass was composed from an ITO cover slip. Both cover slip and slide glass were treated with SE-1211 as described in §2.1. Because ITO absorbs infrared light very well, there was no need to use a heating stage during the operation of the laser tweezers. Also, because high temperatures could be reached at the ITO cover slip's surface, localized melting of the bulk and mesogenic realignment of the substrate were used to bind the upper point singularity of the toron to the polymer-coated SE-1211 substrate, thereby immobilizing the diffusive motion of the toron.

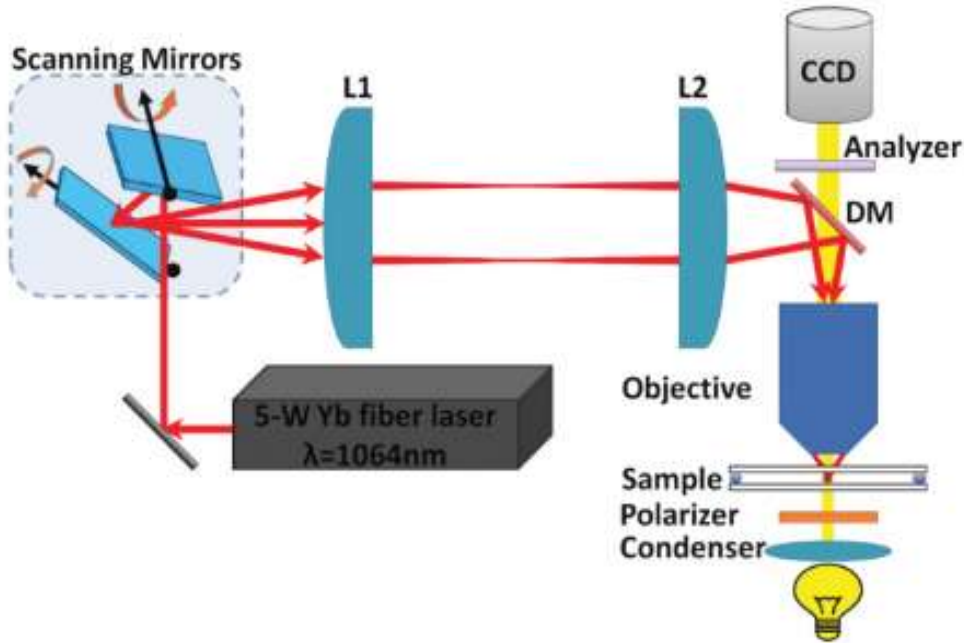


Figure 9: Laser tweezer setup with the 2D position of the beam in the plane of the sample controlled by the angle of two scanning mirrors. Lenses L1 and L2 direct the beam from the CW collimated ytterbium fiber laser into the back aperture of an objective. The dichroic mirror (DM) makes it possible to safely scan the laser and generate director perturbations while simultaneously inspecting those perturbations via the CCD shown above. The crossed polarizer and analyzer are used to obtain polarizing optical microscopy textures. Image adapted from [24].

2.3 Experimental Observation of Nematicons and Topological Solitons with Polarizing Optical Microscopy

In order to image the solitonic director structures so that I could characterize their interactions, I used a homebuilt polarizing optical microscope (POM) shown in Fig. 10. The POM was specially designed to be capable of observing laser light introduced from the side of the cell, *i.e.* light introduced between the pieces of glass as shown below. Exceptional care was taken to ensure parallel alignment of the laser beam and its focal point in the sample with the optics table by calibration against irises of equal height along the beam path (not shown). The position of both the objective tube and the sample could be independently adjusted by two 3-axis micrometer stages. Additionally, the angle of the cell with respect

to the surface of the optics table could be carefully adjusted by optical mount screws in the sample stage.

After careful measurement of the laser light directly emitted from the diode laser's aperture, I found that the light was elipically polarized. I found that the most robust nematicons formed with the elipical long axis at an angle $\sim 60^\circ$ from the vertical x-axis (Fig. 6) but that nematicons would form over a wide range of incident long-axis polarizations from nearly vertically polarized to nearly horizontal. Additionally, while stable nematicons could be generated with as little as ~ 20 mW of power at the sample, I found that a convenient working power was between 25 and 40 mW for the robust formation of stable, thresholdless nematicons.

To observe the nematicons and topological solitons, the homebuilt POM as shown in Fig. 10 was used in a few different modes. First, direct observation of the nematicon's green scattered light was accomplished by removing the wave plate and bandpass filter. However, because the intensity of the scattered light tended to saturate the CCD and obscure the white-light birefringence from topological solitons, a bandpass filter was added to let only red light reach the camera. In this way, both birefringence from the presence of the nematicon and topological solitons could be detected and analyzed. Finally, to understand the director orientation in the nematicon, a wave plate was added along with the bandpass filter.

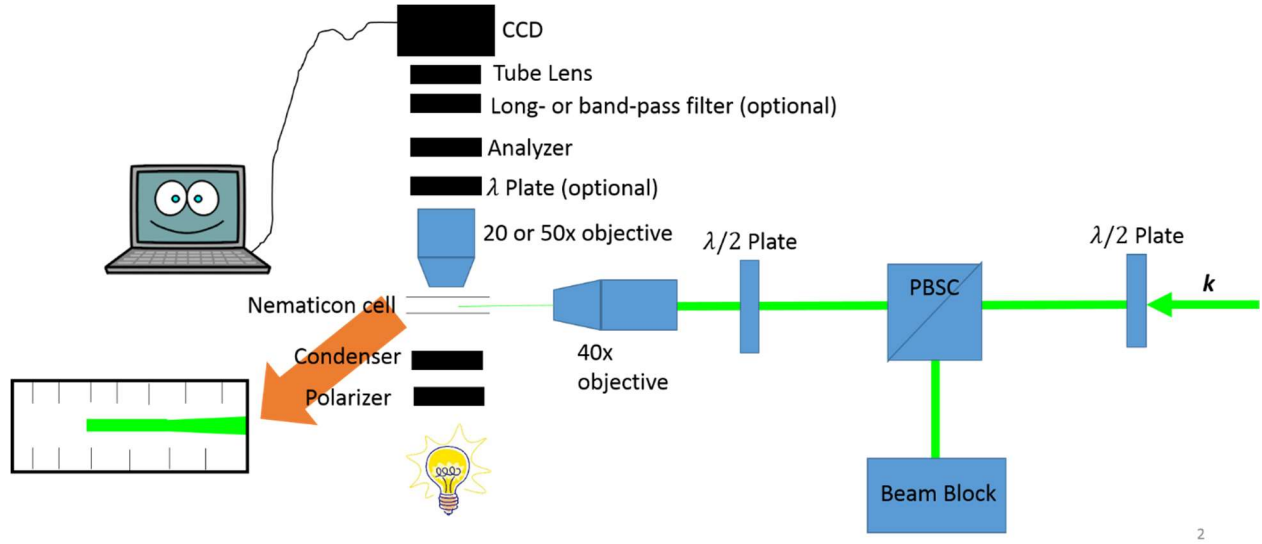


Figure 10: Optical and polarizing optical microscopy observations of elastic interactions between torons and nematicons were obtained with the apparatus shown above. A 532-nm diode laser beam was introduced, as shown with the green arrow denoting the $\mathbf{k}$ vector, to a half-wave plate in order to attenuate the beam's intensity after it passed through the polarizing beam splitting cube (PBSC). A second half-wave plate was used to adjust the incident polarization of the beam. Sufficient intensity to form a thresholdless nematicon was achieved by passing the few-mm wide beam through a 40x objective. Crossed polarizers were used to observe the birefringence of the nematicon with a 26- μm wide bandpass filter (center wavelength 625 μm) to filter out the 532-nm scattered laser light. With the addition of a waveplate, the orientation of the liquid crystal director could be inferred. Interactions were captured with video taken with a CCD.

We may understand the resultant birefringence caused by the director perturbations from the nematicon or topological solitons with the following relationship: $I(\mathbf{r}) = I_0 \sin^2(2\beta(\mathbf{r})) \sin^2 \left[\frac{\pi d}{\lambda_0} (n_e(\theta(\mathbf{r})) - n_o) \right]$ where $I(\mathbf{r})$ is the birefringence intensity at each lateral point $\mathbf{r}$ across the cell, I_0 is the incident intensity on the liquid crystal sample, $\beta(\mathbf{r})$ is the polar angle that liquid crystal molecules make with either the polarizer or analyzer, d is the thickness of the cell, λ_0 is the center wavelength for incident light, $n_e(\theta(\mathbf{r}))$ is the effective refractive index caused by reorienting liquid crystal molecules by an azimuthal angle $\theta(\mathbf{r})$, and n_o is the ordinary index of refraction. (See Fig. 11 below.) Because the primary mechanism behind optical birefringence in liquid crystal in this experiment is phase retardation, the addition of a wave plate allows the determination of the director

orientation (Fig. 12) beyond director orientation down to orthogonal directions, as is possible with only crossed polarizers.

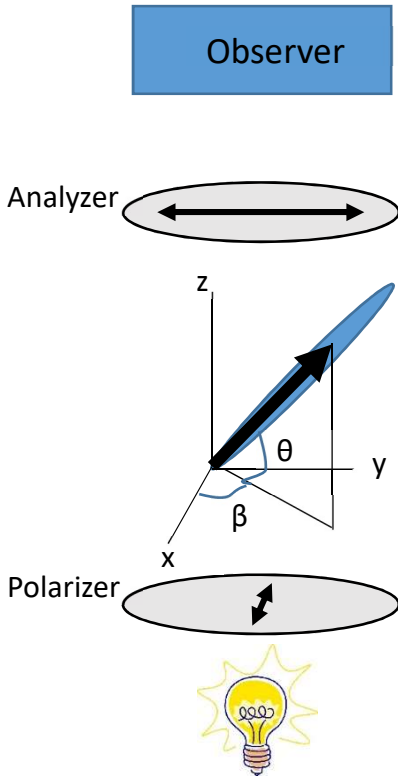


Figure 11: Schematic illustration of liquid-crystal molecular dependence on polar and azimuthal angles for the calculation of intensity from birefringence at the observer.

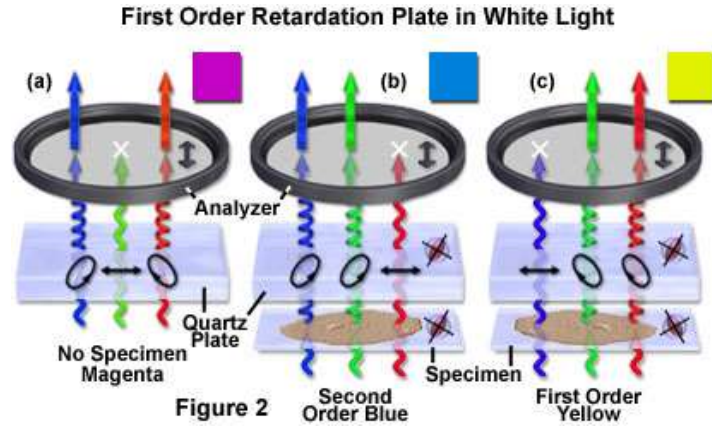


Figure 12: Illustration of the functional principles of a first-order wave plate that retards 532-nm light by 2π . (a) With no sample present, the wave plate (quartz plate) retards green light by 2π such that the linearly polarized green light is blocked by the analyzer while components of the elliptically polarized blue and red light are allowed to pass. (b) The insertion of a birefringent sample causes the wavefront ellipsoids of both the sample and the wave plate to be parallel for red light, which causes the observation of second-order blue. (c) If the sample is rotated 90° , the wavefront ellipsoids are now perpendicular which causes blue light to be blocked for a first-order yellow. Adapted from [26].

In the next subsection, I will discuss how I interpreted data collected using POM.

2.4 Data Analysis

Once I had realized and identified both topological solitons and nematicons in the liquid crystal cells, I attempted to characterize the elastic free energy interaction between the two by determining the force of the interaction between nematicons and torons. The derivation used to determine this force used information from [27]. I start this determination with the Langevin Equation shown in Eq. 1:

$$m\dot{\mathbf{v}} = -\gamma\mathbf{v} + \sigma\xi(t). \quad (1)$$

Where $m\dot{\mathbf{v}}$ is equal to the net force, $-\gamma\mathbf{v}$ is the drag force, and $\sigma\xi(t)$ is the force due to density fluctuations. I make the assumption in this case that $|m\dot{\mathbf{v}}| \ll |\gamma\mathbf{v}|$ because of the viscosity of liquid crystal. This means that I may approximate the acceleration as zero to yield

$$\gamma\mathbf{v} = \sigma\xi(t). \quad (2)$$

In order to find the drag and density-fluctuation coefficients in terms of a known quantity, I can describe the diffusion coefficient in terms of the mean square displacement or variance. The variance may be written as in Eq. 3:

$$\langle \Delta \mathbf{r}^2 \rangle = \int_V dV \Delta \mathbf{r}^2 p(\mathbf{r}, t; \mathbf{r}_o, t_o). \quad (3)$$

where $\Delta \mathbf{r}$ is the displacement, the brackets indicate averaging, p is the probability density of the displacement for an increment between $(\mathbf{r}, t)$ and $(\mathbf{r}_o, t_o)$, where the zero subscripts indicate the starting position and time, respectively, and the integral is taken over all space. I then take the time derivative of the variance:

$$\partial_t \langle \Delta \mathbf{r}^2 \rangle = \int_V dV p(\mathbf{r}, t; \mathbf{r}_o, t_o) \partial_t \Delta \mathbf{r}^2 + \int_V dV \Delta \mathbf{r}^2 \partial_t p(\mathbf{r}, t; \mathbf{r}_o, t_o). \quad (4)$$

The first term of this equation goes to zero to yield

$$\partial_t \langle \Delta \mathbf{r}^2 \rangle = \int_V dV \Delta \mathbf{r}^2 \partial_t p(\mathbf{r}, t; \mathbf{r}_o, t_o). \quad (5)$$

Invoking the Einstein Diffusion equation,

$$\partial_t p(\mathbf{r}, t; \mathbf{r}_o, t_o) = \frac{\sigma^2}{2\gamma^2} \nabla^2 p(\mathbf{r}, t; \mathbf{r}_o, t_o), \quad (6)$$

where σ and γ are the coefficients from the Langevin Equation (1). Substituting Eq. 6 into Eq. 5, I find that

$$\partial_t \langle \Delta \mathbf{r}^2 \rangle = \frac{\sigma^2}{2\gamma^2} \int_V dV \Delta \mathbf{r}^2 \nabla^2 p(\mathbf{r}, t; \mathbf{r}_o, t_o). \quad (7)$$

One can rearrange the integral into an easy-to-solve form by applying Green's Theorem,

$$\int_V dV (u \nabla^2 v - v \nabla^2 u) = \int_S dA (u \nabla v - v \nabla u), \quad (8)$$

where I identify u to be $\Delta \mathbf{r}^2$ and v as $p(\mathbf{r}, t; \mathbf{r}_o, t_o)$. The surface integral on the right goes to zero because the probability density is zero at infinity. Whence,

$$\frac{\sigma^2}{2\gamma^2} \int_V dV (\Delta \mathbf{r})^2 \nabla^2 p(\mathbf{r}, t; \mathbf{r}_o, t_o) = \frac{\sigma^2}{2\gamma^2} \int_V dV p(\mathbf{r}, t; \mathbf{r}_o, t_o) \nabla^2 (\Delta \mathbf{r})^2. \quad (9)$$

On the right side of Eq. 9, $\nabla^2 (\Delta \mathbf{r})^2$ is equal to $2d$ where d is the dimensionality of the position vector $\mathbf{r}$.

Substituting Eq. 9 into Eq. 7, I find that

$$\partial_t \langle \Delta \mathbf{r}^2 \rangle = \frac{\sigma^2}{2\gamma^2} 2d \int_V dV p(\mathbf{r}, t; \mathbf{r}_o, t_o). \quad (10)$$

Assuming that the probability density is a normalized quantity and that the volume integral in Eq. 10 is over all space, the integral goes to unity. When I integrate both sides of the equation with respect to time, I find

$$\langle \Delta \mathbf{r}^2 \rangle = \frac{\sigma^2}{2\gamma^2} 2d \Delta t. \quad (11)$$

The ratio $\frac{\sigma^2}{2\gamma^2}$ is identified as the diffusion coefficient D . Thus, I have written the variance in terms of the diffusion coefficient:

$$\langle \Delta \mathbf{r}^2 \rangle = 2Dd\Delta t. \quad (12)$$

When one considers the diffusion of a particle due to Brownian motion without any external applied force, the resulting diffusion can be described by a Wiener process. This means that the total probability can be described by

$$P(\mathbf{r}, t; \mathbf{r}_o, t_o) = Ae^{-\frac{(\Delta r)^2}{4D\Delta t}} \quad (13)$$

where A is a constant. With substitution of Eq. 12 into Eq. 13, I find that

$$P(\mathbf{r}, t; \mathbf{r}_o, t_o) = Ae^{-\frac{(\Delta r)^2 d}{2(\Delta r^2)}}. \quad (14)$$

Using Eq. 14, one can find the variance by observing a particle's trajectory due to Brownian motion and by fitting the probability distribution of that particle's trajectory with Eq. 14. First, I took a video of an isolated toron with no external forces acting on it to capture its diffusive motion for ~10-30 minutes. After tracking the toron's lateral coordinates frame-by-frame with video analysis software [28, 29], I then created two 1D histograms along each lateral axis of the displacement of the toron between consecutive frames. Because there is no correlation between motions along each axis, the 2D analysis problem may be simplified to two 1D problems. Because I know that each 1D probability distribution of this random walk motion can be modeled with Eq. 14, I then used least-squares fitting to fit the histogram with the function $P(r) = Ae^{-\frac{(r-B)^2}{2\langle r^2 \rangle}}$, where B is another constant. (See Fig. 13 below.) When completed, I obtained values for the mean square displacement along each lateral axis.

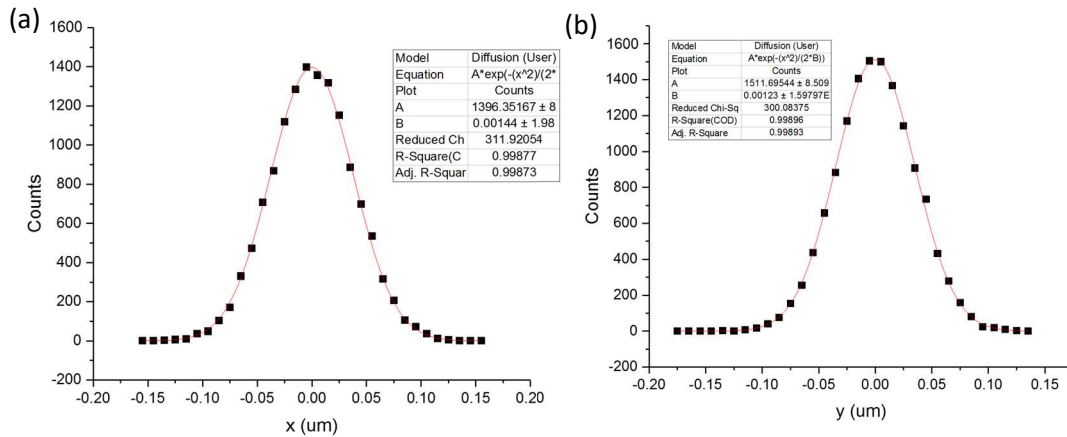


Figure 13: (a) Examples of diffusion histogram steps along x axis (black points). Gaussian least-squares fit (red line) provides the mean square displacement, a quantity which is used to calculate the elastic potential energy of the elastic interaction. (b) Histogram of diffusion steps along the y -axis.

To find the elastic interaction force on the toron, I needed to invoke the Einstein-Smoluchowski Relation:

$$\mathbf{F}_{app} = \frac{k_b T \mathbf{v}}{D} \quad (15)$$

where k_b is the Boltzmann constant, T is the ambient temperature, and $\mathbf{v}$ is the drift velocity of the toron. Substituting Eq. 12, with $d = 1$, into Eq. 15, I find for the 1D case that

$$F_{app} = \frac{2k_b T \Delta t}{\langle (\Delta r)^2 \rangle} v. \quad (16)$$

In order to find the drift velocity, I tracked the position of the toron, using video analysis software, as it moved away from the nematicon. (See Fig. 14(a).) Though I did have a displacement step between each video frame, finding the velocity from the raw data by dividing the displacement by time step created an extremely noisy velocity signal because of the intrinsic Brownian motion of the toron. I used least-squares fitting to generate a polynomial fit of arbitrary order (less than 30th order) for my displacement data in both x and y dimensions as a function of time. (See Fig. 14 (b, c).) I determined the order of the polynomial by inspection of the velocity curves and the agreement of their antiderivatives' slopes with their fits to the raw position data. I took the derivative of that polynomial to find our velocity components.

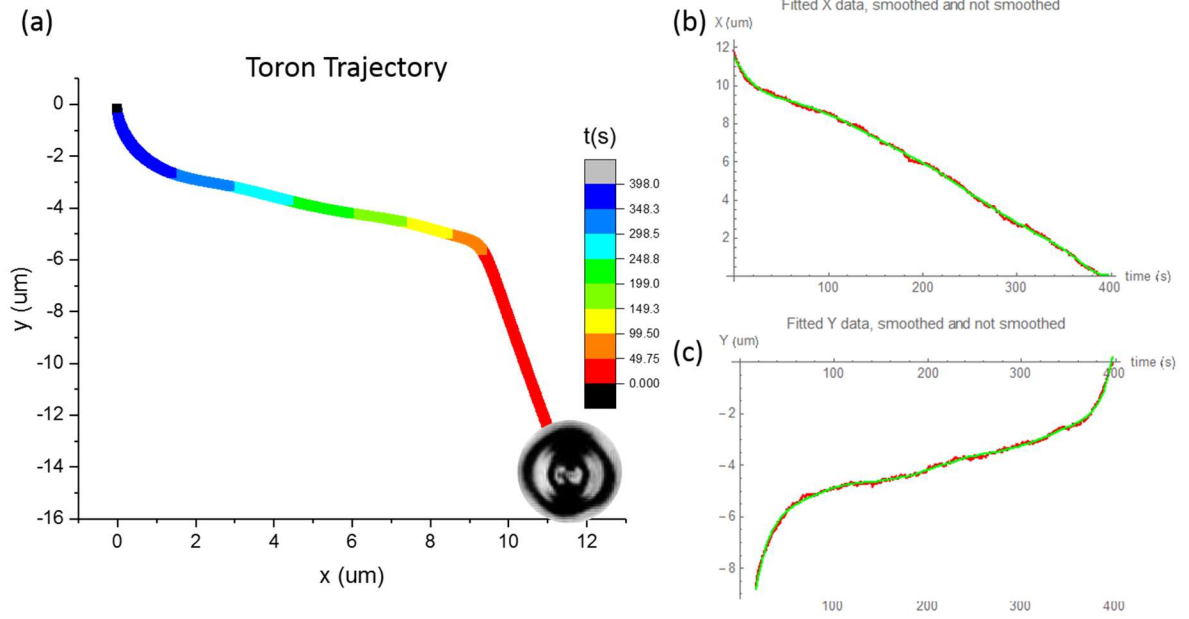


Figure 14: (a) The trajectory of the toron as determined from the fitted polynomials to the raw position versus time data in (b) and (c). The colors along the trajectory represent the various times at which the toron was at each point. The example image of a toron is not to scale. (b) Red raw x-position data collected with video analysis software was fitted with a polynomial (green curve) of arbitrary order. (c) Y-position data was also fitted with a polynomial.

Finally, I substituted the velocity components into the formula for elastic force to calculate the elastic force on the particle at each point along its trajectory. By using the relationship $U = \sum_i |\mathbf{F}_i| |\Delta \mathbf{s}_i|$, which is a discrete sum for the elastic potential energy, and starting the sum from the last coordinate point, I was able to calculate the work required to bring the toron to each point along the toron's trajectory. (The force $\mathbf{F}_i$ is always parallel to the displacement $\Delta \mathbf{s}_i$.) (See Fig. 15 below.) I took a full population of nematicon trajectories where the toron started at several different points along the nematicon beam path and completed the process described above for each trajectory. The resultant contour maps are presented in §3.

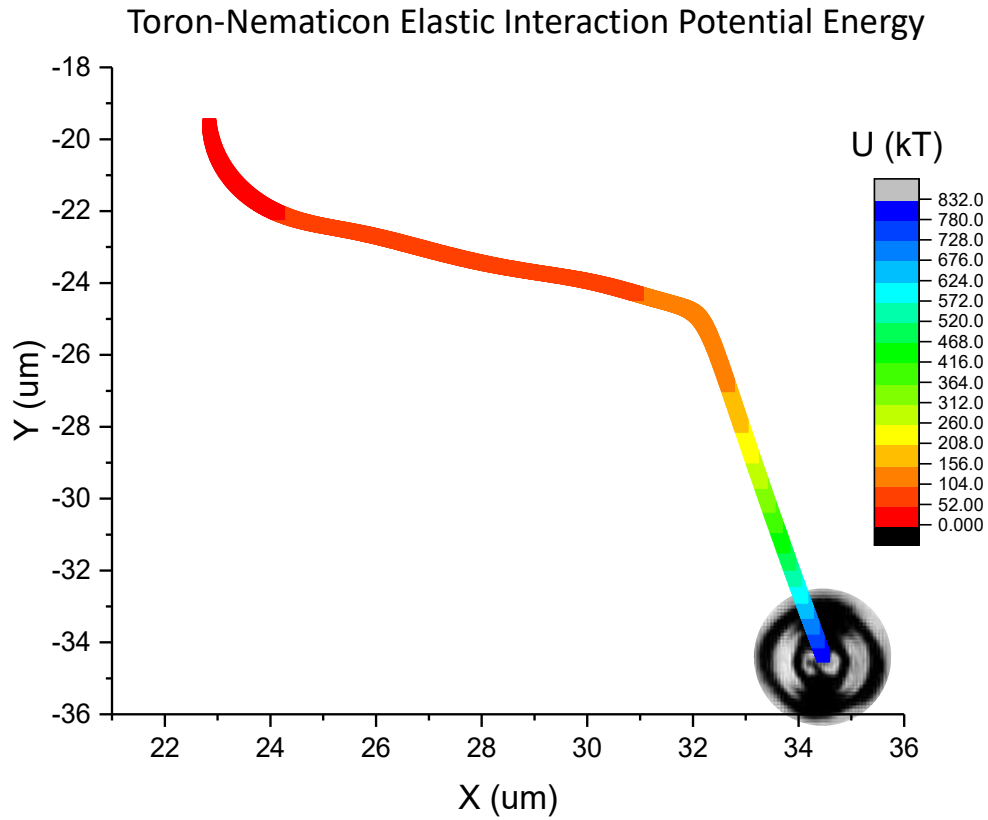


Figure 15: The trajectory of the toron as determined by polynomial fits of raw x- and y-data. The elastic potential energy at each point along the trajectory is given in multiples of kT .

3. Experimental Results

In this section, I will discuss the results achieved using the methods described in the previous section. I will review my success in generating both cholesteric fingers and torons as well as undulating thresholdless nematicons. After discussing the successful generation of both of these features, I will delve into their interaction. I observed and analyzed the elastic interaction between a toron and a stationary nematicon as well as the deflection of a nematicon by a stationary or pinned toron.

3.1 Generation of Topological Solitons

Using the experimental methods described in §2.2, I was able to generate arbitrary arrays of both torons and cholesteric fingers, as shown in Fig. 16 below. I chose the shapes of the toron arrays and cholesteric fingers because of the potential for interesting interactions with nematicons. For example, in Fig. 16(a), a crystallographic defect exists in a 2D crystal of torons. I wanted to test if the nematicon would be split in two at the defect. Additionally, the curved linear array of torons and the fingers in Fig. 16 (b - d) have the potential to guide and deflect the nematicon via elastic interactions.

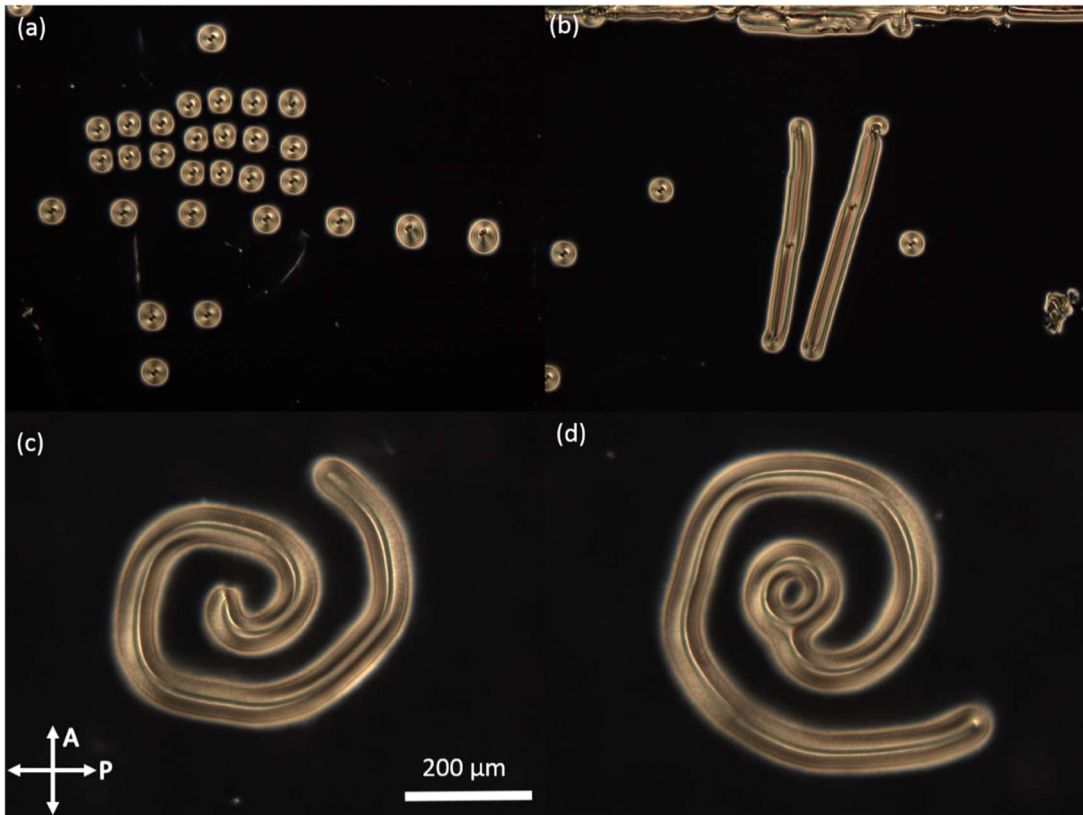


Figure 16: Topological solitons. (a) Pinned arrays of torons configured in a 2D crystal with crystallographic defect and a curving linear array. Mobile torons exist above and below the pinned arrays. (b) Fingers with pinned terminal points. Mobile torons exist on either side. Both (a, b) were generated in a $\sim 16\text{-}\mu\text{m}$ -thick cell. (c) Cholesteric finger with clockwise spiraling. (d) Cholesteric finger with anti-clockwise spiraling and center loop. Both (c, d) were generated in a cell $\sim 30\text{-}\mu\text{m}$ thick. All images were taken with crossed polarizers as indicated with the arrows.

3.2 Generation and Analysis of Nematicons

I was also able to generate nematicons in a chiral nematic liquid crystal cell using the methods described in §2.3. This is significant because, while nematicons have been generated in homeotropically aligned nematic liquid crystal before, this is the first work to generate nematicons in *chiral* nematic liquid crystal. The scattered light image of this nematicon is shown in Fig. 17 (a). There is a clear periodic beating pattern shown in this image. It has been reported that a periodic beating pattern is both expected from theory and observed in thresholdless nematicons within homeotropically aligned nematic liquid crystal cells; however, the periodicity of the beat pattern observed in Fig. 17 (a) does not align with the predicted periodicity described in section §1.3. Observation of the birefringent texture of the nematicon, as is shown in Fig. 17 (b-c), provides evidence for a helicoidal twist of the director about the propagation axis of the nematicon. Upon rotation of crossed polarizer and analyzer (not shown), I observed that the bright spots along the propagation axis of the nematicon shown in Fig. 17 (b) translate along the propagation axis. Coupled with the thresholdless condition (a superposition of TE and TM modes) for generating the nematicon, translation of the bright birefringent texture indicates director distortion not only in the lateral plane of the sample but also in the vertical plane, *i.e.* a helicoidal twist about the propagation axis. Finally, Fig. 17 (c) provides sound evidence that the helicoidal twist periodicity is equal to the electromagnetic beat periodicity, as shown in Fig. 17 (a). Interestingly, neither electromagnetic beating nor birefringent texture periodicity agree with the simple beat periodicity relationship introduced in §1.3 for homeotropic nematic systems. In fact, I observed that the periodicity increases as beam power increases, an observation which disagrees with the homeotropic nematic beat relationship. I propose that the helicoidal twist tendency of frustrated chiral nematic liquid crystal modifies the behavior of nematicons launched in such systems to realize the behavior that I observed.

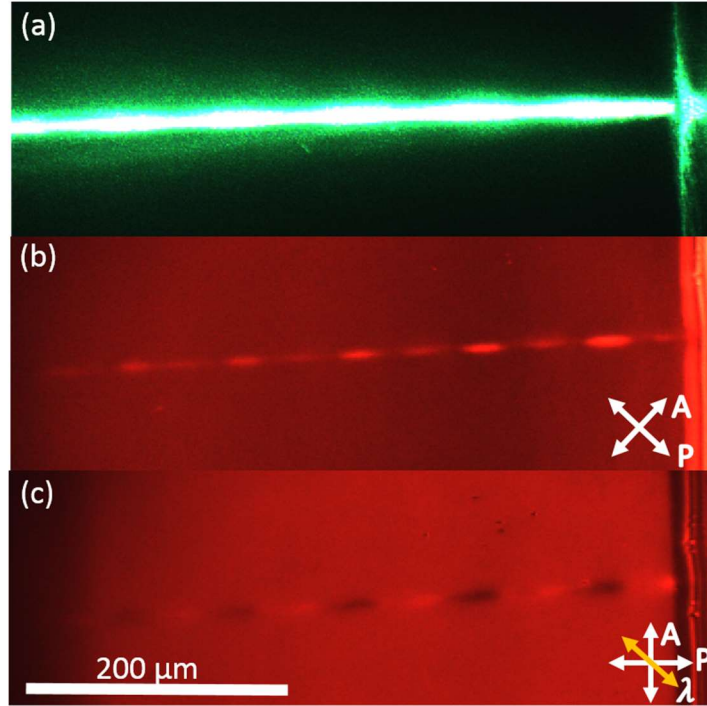


Figure 17: Nematicon in frustrated chiral nematic liquid crystal. All images were taken with ~ 26 mW of nematicon power. (a) Scattered green light image of nematicon. Note undulating structure which is indicative of a superposition of TM and TE modes for thresholdless nematicons. (b) Birefringence of nematicon observed through a bandpass filter. The bright and dark beats correspond to the twisting of the chiral nematic liquid crystal along the propagation axis. (c) Wave plate analysis of nematicon with bandpass filter with slow axis of wave plate as indicated with the arrow. Dark and bright regions correspond to the alignment direction of the director within the nematicon.

3.3 Elastic Interactions between Stationary Nematicons and Mobile Toron

Because it is energetically favorable for the director near, but not within, the toron to be homeotropically aligned, the system will try to move toward a state where this is the case. Since the nematicon distorts the director away from this homeotropic alignment, and because the nematicon is stationary and stable (non-filamenting or -undulating), it is likely that the toron will translate its lateral position so that the director near it will once again be homeotropically aligned. Additionally, transient beam deflection has been observed with this type of interaction. This type of interaction, in which a

toron will move away from a distortion in the director, has been observed when two toron are placed near one another [30]. This toron-nematicon interaction is shown in Fig. 18.

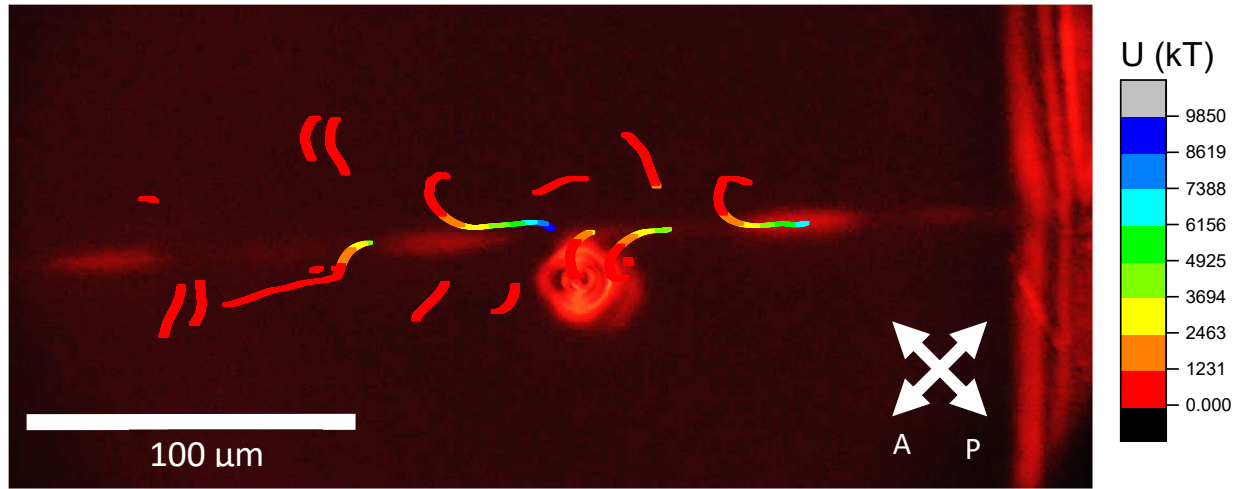


Figure 18: Toron interaction trajectories plotted with color-coded elastic potential energy along nematicon propagation axis.

Using the particle tracking procedure described in §2.4 and connecting the points of equal elastic potential energy along the trajectories shown in Fig. 18, I produced the elastic potential energy contour plot that is shown in Fig. 19. Note the clear correspondence of scattered light beat periodicity (Fig. 17(a)) and director twisting periodicity (Fig. 17(c)) with the peaks in the potential energy contour map.

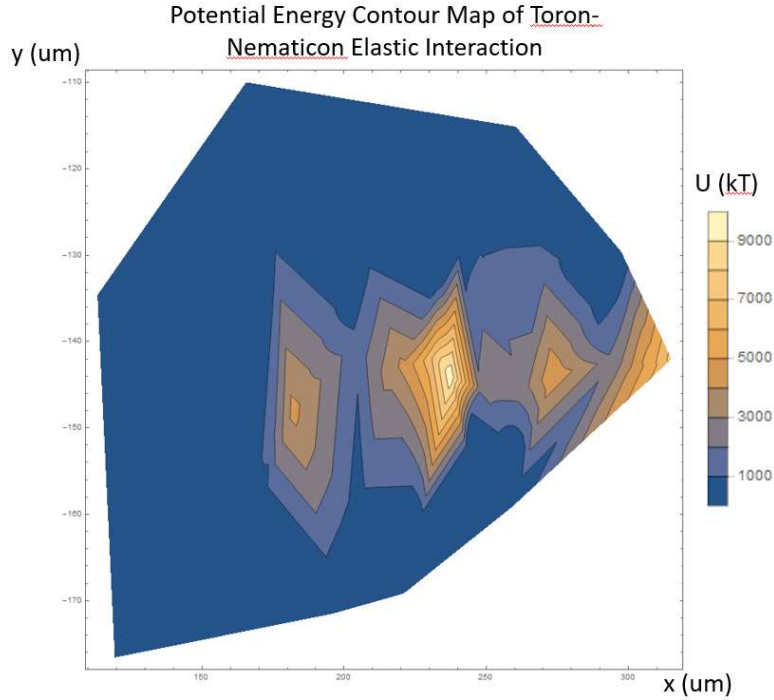


Figure 19: Elastic potential energy contour plot constructed from interaction paths and equi-potential energies along toron trajectories.

From this graphic, it is clear that the beating of the nematicon has an effect on the strength of the toron-nematicon interaction. The toron moves much more slowly, and thus has a much lower potential energy, when it is near a region of destructive interference in the nematicon than it does when it is near a region of constructive interference (Fig. 17(a)). Fig. 19 was generated using approximately 15 different interactions.

3.4 Nematicon Deflection with Stationary Topological Solitons

Conversely, if a topological soliton is fixed in place, that is, torons are pinned on a glass substrate such that they cannot translate, steady-state beam deflection is observed. This occurs because the addition of a new reorientational force changes the lensing near the toron. A minimization of the Frank-Oseen free energy for this interaction establishes a minimum when the nematicon is deflected from the toron, as shown in Fig. 20. I show the deflection of a nematicon beam in a cell with a

thickness-to-pitch ratio of ~ 0.9 . First, I deflected the nematicon beam using a cholesteric finger at multiple points along the finger (Fig. 20(a, b)). Additionally, I deflected the nematicon beam using arrays of torons identical to those in §3.1. The arrays of torons guided the nematicon along the curved paths which deviate from the unperturbed nematicon trajectory. Because of this, I can see from Fig. 20 (c, d) that it is possible to provide very specific redirections of the nematicon using arrays of stationary torons.

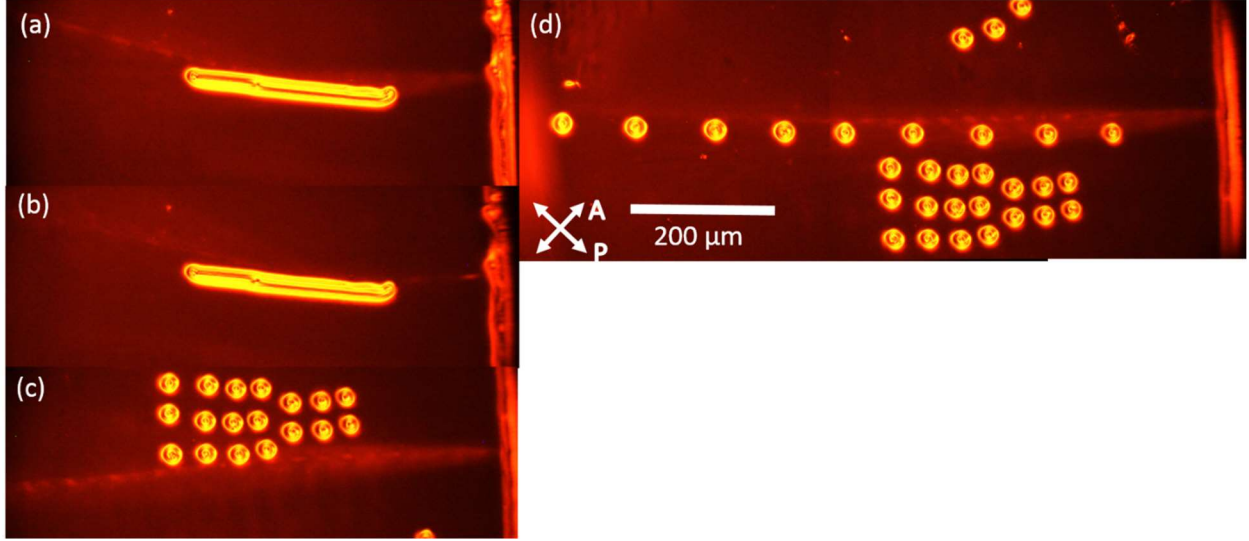


Figure 20: Nematicon deflection from stationary topological solitons. (a, b) Nematicon deflected from a cholesteric finger pinned at its ends. (c) Nematicon deflected and guided by exterior of 2D toron crystal. (d) Guiding of the nematicon along a curved linear array of torons. All images taken with a bandpass filter and under crossed polarizers as indicated by the arrows with a cell thickness of $\sim 16 \mu\text{m}$.

These results indicates that I can achieve very specific redirection of nematicons using topological solitons, with extension to the application as a possible tool for all-optical telecommunications.

4. Conclusion

The interaction between topological solitons and nematicons in chiral nematic liquid crystal is interesting on both theoretical and application levels. In this work, I describe the two phenomena in

isolation, with introductory context. I then discuss the experimental methods that I used to create liquid-crystal cells in which we could test their mutual interaction via distortions of the director, the way that I realized both of these phenomena in those liquid-crystal cells, and the way that we imaged and analyzed each phenomenon and their interaction. Finally, I discussed the conclusions we can draw from the results we gathered. The first conclusion is that a helicoidal twist along the propagation axis of a thresholdless nematicon generated in a homeotropically aligned chiral nematic liquid crystal cell exists. I postulate that the twist alters the expected periodic beat structure of the nematicon. Additionally, I determined that there is an interaction which causes diffusive topological solitons to move away from that same nematicon and that the strength of that interaction is correlated to the periodic beat structure of the nematicon. Finally, I determined that pinned topological solitons in these cells will deflect the nematicon beam.

The possible applications for real-time beam deflection of nematicons without the use of external voltages or control lasers are promising. Other methods like the one described in [2-8] are promising, but it has the drawback of being power-intensive, in that it requires one or two control lasers to be inserted in the sample at all times, or permanent. Other methods of nematicon deflection (§1) have different drawbacks. With the new methods presented in this paper, it will be possible to achieve the goal of beam deflection using entirely erasable and non-power-intensive topological solitons. Possible applications of a stationary nematicon moving a toron also abound. Beyond the fact that this novel interaction has never before been observed, which makes its characterization interesting in its own right, the translation of torons by their deflection from nematicons may be pre-configured for transient nematicon guidance. The pre-configurability of this system opens up possibilities such as mobile logic gates which perform irreversible data manipulation, which may be useful as cryptographic keys, as one example.

Though this experiment did provide novel insight into nematicons in frustrated chiral nematic liquid crystals and their interactions with topological solitons, there are two major areas of experimentation that I would like to pursue further. The first is that I would like to include a colloidal dispersion of ferromagnetic nanoplatelets in the liquid crystal to control the size of torons and cholesteric fingers with the application of an external magnetic field. By modulating the direction of the applied field, torons and cholesteric fingers can be pre-engineered to grow or shrink [31]. In this manner, the deflection of the nematicon from these structures can be controlled to form all-optical logic gates with facile pre-configurability. Beam deflection with nematicons has already been used to create all optical logic gates, but this method has the advantage, described earlier, of being impermanent and not power intensive[2-8]. Finally, I would also like to carry out simulations of the director structure during both the optical and elastic interactions to better understand our empirical data. The Smalyukh Group has already created in-depth simulations of the director structure of topological solitons which minimize the free energy in a frustrated chiral nematic cell [15]. It will be possible to alter this simulation so that it can also include a free energy minimization for a stationary nematicon and the interaction between a nematicon and a topological soliton.

5. References

- [1] B. Bahadur, *Liquid crystals: applications and uses* (World Scientific, Singapore, 1995).
- [2] Y.V. Izdebskaya, A.S. Desyatnikov, et. al. *Opt. Soc. Am. B* **30**, 1432 (2013).
- [3] Piccardi, A., Peccianti, M., et.al. (2009). *Applied Physics Letters*,94(9)
- [4] A. Piccardi, A. Alberucci, et. al. *Applied Physics Letters* 96, 071104 (2010).
- [5] Kwasny, M., Piccardi, et. al. (2011). *Optics Letters*,36(13), 2566
- [6] Piccardi, A., Alberucci, et al. (2012). *Applied Physics Letters*,100(25), 251107.
- [7] A. Alberucci, M. Peccianti, et. al. *Optics Letters* 32, 2795 (2007).
- [8] A. Piccardi, A. Alberucci, et al. *Advanced Photonics & Renewable Energy* (2010).
- [9] L., *Chemistry LibreTexts* (2016).
- [10] G. Vertogen, *Thermotropic liquid crystals, fundamentals* (Springer, 2012).
- [11] Liquid Crystals: a Simple View on a Complex Matter.
<http://www.personal.kent.edu/~bisenyuk/liquidcrystals/>
- [12] W.A. Dowden, *Non-Destructive Testing* 1, 99 (1967).
- [13] Liquid Crystal (LC) Materials | TCI America. (n.d.). Retrieved April 03, 2017, from
http://www.tcichemicals.com/eshop/en/ns/category_index/12775/
- [14] G. Assanto, *Nematicons- Spatial Optical Solitons in Nematic Liquid Crystals* (Wiley, Hoboken, NJ, 2013).
- [15] P.J. Ackerman, et al. *Phys. Rev. E* **86**, 021703 (2012)

- [16] I.I. Smalyukh, B.I. Senyuk, et. al. *Physical Review E* **72**, (2005).
- [17] M. Peccianti and G. Assanto, *Physics Reports* **516**, 147 (2012).
- [18] M. A. Karpierz, *Phys. Rev. E* **66**, 036603 (2002)
- [19] E. Braun, L. P. Faucheux, et al., *Phys. Rev. A* **48**, 611 (1993)
- [20] M. Warenghem, J. F. Henninot, and G. Abbate, *Opt. Express* **2**, 483 (1998); *Mol. Cryst. Liq. Cryst.* **320**, 207 (1998)
- [21] M. A. Karpierz, M. Sierakowski, et al, *Mol. Cryst. Liq. Cryst.* **320**, 157 (1998)
- [22] M. A. Karpierz, M. Sierakowski, et al, *Mol. Cryst. Liq. Cryst.* **320**, 157 (1998)
- [23] M. Peccianti, A. De Rossi, et al., *Appl. Phys. Lett.* **77**, 7 (2000)
- [24] M. Peccianti and G. Assanto, *Opt. Lett.* **26**, 1690 (2001); **26**, 1791 (2001); *Phys. Rev. E* **65**, 035603 (2002)
- [25] Olympus Microscopy Resource Center | Polarized Light Microscopy - The First Order (Full Wave) Retardation Plate. <http://www.olympusmicro.com/primer/techniques/polarized/firstorderplate.html>
- [26] Schulten, K. (n.d.). Theoretical Biophysics. Retrieved April 14, 2017, from <http://www.ks.uiuc.edu/Services/Class/PHYS498/>
- [27] National Institutes of Health. <https://imagej.nih.gov/ij/notes.html>
- [28] C.I. Nussbaum-Krammer, M.F. Neto, et. al. *Journal of Visualized Experiments* (2015).
- [29] Boyle, Timothy J., "Characterizing the Dynamic Properties of Toron-Umbilic Pairs" (2016). Undergraduate Honors Theses. Paper 1057.q
http://scholar.colorado.edu/cgi/viewcontent.cgi?article=2397&context=honr_theses

[30] Zhang, Q., Ackerman, P. J., et. al. (2015). Ferromagnetic Switching of Knotted Vector Fields in Liquid Crystal Colloids. *Physical Review Letters*, 115 (9).

[31] M. Peccianti, C. Conti, et. al. Applied Physics Letters 81, 3335 (2002).