

# **Efficient Synthesis of Network Updates**

by

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Efficient Synthesis of Network Updates

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Software Defined Networking simplifies network control, configuration and setup by abstracting some steps in these processes. Packet traffic routes in a Software Defined Network(SDN) may need to change due to policy changes. To implement these policy changes, we need to ensure that the SDN is updated consistently. Any packet in the network should be routed either according to the new policy or the old policy. If a packet can take a route which lies partially in the old policy and partially in the new, the network is said to be in an inconsistent state. We present an algorithm that produces an order of node updates which preserves consistency in an SDN. This algorithm will either find an order that preserves efficiency in the SDN or fail stating that such an order does not exist for the SDN.

When a node is updated, its routing tables are changed. However, it may take some time for the network to experience this change as there may be some slow packets on the old routes outgoing from it. For this reason, we may need to wait before we update the next node. Our algorithm finds a consistency preserving order that needs minimum number of waits.

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## Chapter 1

### Introduction

Software Defined Networking(SDN) is revolutionizing the networking industry, by abstracting low level networking tasks and providing high level APIs for network programmers. However, there are still very few mechanisms that reliably abstract the updating of global configurations. Even if initial and final configurations are correct, naïvely updating individual nodes can lead to incorrect transient behaviors, including loops, black holes, and access control violations. In this study, we present an approach for automatically synthesizing updates that are guaranteed to preserve consistency properties with minimal wait time throughout the update. We ensure per-packet consistency [9], a guarantee that every packet traversing the network will follow one global configuration, either old or new, throughout the update. Every packet traverses a route specified either by the policy in place prior to the update or by the updated policy and not a mixture of both. This would ensure that there are no transient loops or blackholes in the network as long as there are none in the initial and final network configurations.

To reliably update all nodes on the network, we may need to pause the update mechanism(wait) after an update, in order to let traffic along paths that were removed get flushed from the network. In Chapter 5, we shall see that a wait may not be required after every update. We find an order of updates such that the number of waits required is minimal.

The network in our model is a directed graph, with non-weighted edges, and we update it from an initial configuration, to a final configuration, with minimal waits, while preserving per-packet consistency throughout the update. Such an order may not always exist, in which case, we state

that a consistency preserving order does not exist. Other papers have presented search-based approaches [4, 3, 5, 11, 8, 7, 10], that deploy models to prune search trees based on heuristics and constraints for preserving consistency. We present a polynomial time algorithm that updates nodes sequentially, and whenever possible, avoids waiting after an update.

Chapter 2 presents a few examples of consistent network updates which illustrate the challenges for finding a consistent order of updates. Chapter 3 formalizes our network model, presents precise definitions of consistency and waits, and states our problem statement. Chapter 4 presents a polynomial time algorithm, `OrderUpdate`, which finds an order of updates that preserves consistency. In this chapter, we also prove the correctness and completeness of the `OrderUpdate` algorithm. Chapter 5 presents a modification, `PickAndWait`, to the `OrderUpdate` algorithm that minimizes the number of required waits. This modification adds a degree of determinism to the non-deterministic `OrderUpdate` algorithm using a greedy scheme. We prove that the `OrderUpdate` algorithm with the `PickAndWait` modification, is complete, and produces an order of consistent updates with minimal number of waits. We finally discuss related work in Chapter 6 and conclude our results in Chapter 7.



## Chapter 2

### Examples

Let us consider some easy cases. In Figure 2.1 and Figure 2.2,  $C_i$  edges are solid and  $C_f$  edges are dashed. In Figure 2.1, there is a trivial update order - A,H1,B. Note that H2 does not need to be updated as it has no outgoing edges. However, in Figure 2.2, no matter the order you update nodes in there will always be inconsistency. This is because  $H_1$  can not be updated unless downstream path from  $C$  to  $H_2$  is updated. But also,  $C$  can not be updated unless the upstream path from  $H_1$  to  $C$  is updated. We refer to this case as the Double Diamond case. There is a circular dependency between  $H_1$  and  $C$ .

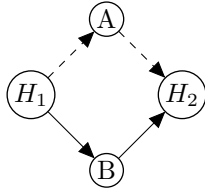


Figure 2.1: A trivial update

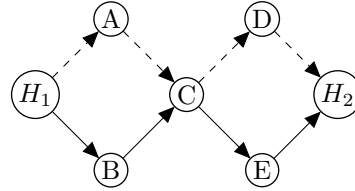


Figure 2.2: Double Diamond case. No update sequence exists.

However, we can not say that the presence of a double diamond implies that there can not be a solution. In Figure 2.3, there is a double diamond between  $D$  and  $G$ (there are mutiple double diamonds in this figure), but updating  $B$  and waiting removes the old traffic incoming to  $D$ . The nodes  $D, E, G, F, H, I$  and  $J$  have no incoming traffic. These disconnected nodes can be updated without worrying about consistency. So, the circular dependency is removed. A valid update order(not considering waits) would be  $A, H_1, K, L, B, D, E, F, G, H, I, J, C, M$ . So we see that it is

not trivial to know whether an update order exists, and if it exists, to find it.

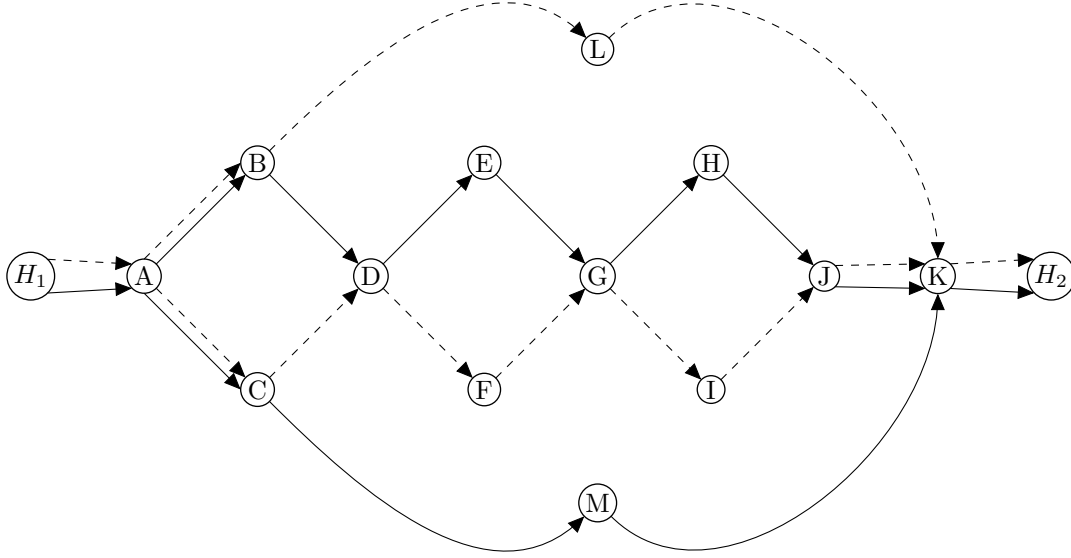


Figure 2.3: Removable Double Diamond. Here  $C_i$  edges are solid and  $C_f$  edges are dashed.

These examples lead us to believe that we need a systematic algorithm to find out whether a consistency preserving update order exists, and if it does, then find it. We shall present an algorithm that does this and also prove its correctness and completeness.

## Chapter 3

### The Network Model

**Network and Configurations.** A topology of the network graph  $G$ , is a tuple  $(N, E)$ , where  $N$  is a set of nodes; and  $E$  is a given set of directed edges. The configuration,  $C \in \mathcal{P}(E)$ , of a network  $G$  is the set of edges present in  $G$  at some time instant. Updating a node removes some old edges and adds new edges, thus changing the configuration of  $G$ . Our goal is to bring the network, from an initial configuration  $C_i$ , to a final configuration  $C_f$ . Random is a function  $\mathcal{P}(N) \rightarrow N$ , that given a set of nodes  $P$ , picks a random node  $a = \text{random}(P)$  from it.

**Sources and Sinks.** The directed graph  $G$ , in any configuration, has only one source  $H_1$  and one sink  $H_2$ .

**Cycles.** The graph  $G$  is acyclic in any configuration. Cycles in  $G$  are undesirable as it would mean that traffic can loop forever in the network.

**Updates.** Let  $R$  be the set of all sequences that can be formed using nodes in  $N$  without repetition. To update a graph  $G$ , we define  $upd$  as a function  $\mathcal{P}(E) \times R \rightarrow \mathcal{P}(E)$ , that given a configuration  $C$  and a sequence of nodes  $S$ , outputs an updated configuration  $C' = upd(C, S)$ .

**Paths.** Let  $Q$  be the set of all possible directed paths in network  $G$ . For obtaining paths, we define  $paths$  as a function  $N \times N \times \mathcal{P}(E) \rightarrow \mathcal{P}(Q)$ , that given a start node  $s$ , an end node  $t$ , and a configuration  $C$ , outputs a set of all paths  $P = paths(s, t, C)$  between  $s$  and  $t$  in configuration  $C$ . We define the  $\in$  operation for a path  $p$  and configuration  $C$  so that, if  $p \in C$ , then all edges in path  $p$  lie in set  $C$ .  $nodes$  is a function  $Q \rightarrow \mathcal{P}(N)$ , that given a path  $q$  returns a set  $S = nodes(q)$  of all nodes on a path.

**Consistency.** A configuration  $C$  is a consistent configuration iff  $\forall p \in \text{paths}(H_1, H_2, C) : p \in C_i \vee p \in C_f$ . There are no paths between  $H_1$  from  $H_2$  that satisfy neither the old policy nor the new policy. This means that if an edge in  $C$  lies only in  $C_f(C_i)$  then all paths in  $C$  that include this edge must lie in  $C_f(C_i)$ .

**Packets.** We assume that all packets in the network are of a single type. This assumption lets us focus on consistency issues which occur due to the network graph alone.

**Waits.** Let  $C_c = \text{upd}(C_i, U)$  be the current configuration reached after updating a sequence  $U$ . Let  $C_w = \text{upd}(C_i, U')$ , where  $U'$  is a prefix of  $U$ , be an intermediate configuration which was reached while updating sequence  $U$ . And we chose to update some node  $n$ , s.t.  $\exists p \in \text{paths}(H_1, n, C_w) : p \notin C_c \wedge \exists q \in \text{paths}(n, H_2, \text{upd}(C_c, n)) : q \in C_f$ , then if there were no waits between  $C_w$  and  $C_c$ , we need to wait before updating  $n$ . This wait is required because some old configuration had a old path which was removed (only  $C_i$  paths can be removed). So, traffic along this removed path needs to be flushed as there is a  $C_f$ -only path downstream after the update. Not waiting would send  $C_i$  traffic on a  $C_f$  path, resulting in inconsistency.

**Commands.** Our update mechanism consists of two commands - update and wait. The update command updates a specified node and changes the configuration of  $G$ . Since  $|N|$  nodes are updated, the number of update commands is always a constant. The wait command simply pauses the update mechanism for some time to allow packets along old edges to get flushed from the network. The wait command does not change the configuration of  $G$ .

**The Network Synthesis Problem.** We need to find a sequence of commands  $U = v_1, v_2, \dots, v_n$  such that:

- (1) After executing the sequence  $U$ ,  $G$  is in configuration  $C_f$ .
- (2) Configuration of  $G$  after executing each command is consistent.
- (3)  $n$  is minimal.

Or state that such a  $U$  does not exist.

## Chapter 4

### The OrderUpdate Algorithm

#### 4.1 Conditions for updating nodes

At any point of time during the update, let us refer to the intermediate configuration at this time as  $C_c$ , denoting current configuration. We shall assume that  $C_c$  is consistent and we find a node  $s$  to update such that the new configuration  $upd(C_c, s)$  is also consistent. Since  $C_c = C_i$  initially, this assumption is correct for the first node update. And so, starting from  $C_i$ , at every point of time, we find a node to update so that the updated configuration is also consistent. In every intermediate configuration  $C_c$ , if a node  $s$  follows conditions in Figure 4.1, we can update it. We now explain why these conditions are necessary for getting an order of updates that maintains consistency. To update a node, it needs to satisfy an upstream and a downstream condition of any one of the types listed below. In every case, there is an upstream condition on a node that gives us information about the paths that the packets on the network have taken to reach it from  $H_1$ . Based on this information, it may or may not be safe to route the traffic downstream to  $H_2$  according to the new routing policy. For every upstream condition, a downstream condition must be satisfied by a node to be updated. In general, a node that satisfies an upstream condition is called a **candidate** node and if this candidate satisfies the downstream condition, then it is called a **valid** node.

- **Type-A or Disconnected** Nodes - Some nodes in  $C_c$  have no traffic incoming to them.

Updating these nodes does not cause any change in network traffic and thus maintains

Type	Upstream(Condition for $paths(H_1, s, C_c)$ )	Downstream(Condition for $paths(s, H_2, C_c)$ )
A	$Z_a = \neg p \in paths(H_1, s, C_c)$	-
B	$Z_b = \neg Z_a \wedge \forall p \in paths(H_1, s, C_c) : (p \in C_i \wedge p \in C_f)$	$\forall p \in paths(s, H_2, upd(C_c, s)) : p \in C_i \vee p \in C_f$
C	$Z_c = \neg Z_b \wedge \forall p \in paths(H_1, s, C_c) : p \in C_f$	$\forall p \in paths(s, H_2, upd(C_c, s)) : p \in C_f$
D	$Z_d = \neg Z_b \wedge \forall p \in paths(H_1, s, C_c) : p \in C_i$	$paths(s, H_2, upd(C_c, s)) \neq \phi \wedge \forall p \in paths(s, H_2, upd(C_c, s)) : p \in C_i$
E	$Z_e = \neg Z_a \wedge \neg Z_b \wedge \neg Z_c \wedge \neg Z_d = (\exists p_f \in paths(H_1, s, C_c) : p_f \in C_f \wedge p_f \notin C_i) \wedge (\exists p_i \in paths(H_1, s, C_c) : p_i \in C_i \wedge p_i \notin C_f)$	$\forall p \in paths(s, H_2, upd(C_c, s)) : p \in C_i \wedge p \in C_f$

Figure 4.1: Necessary conditions for updating a node  $s$ 

consistency. There is no distinction between a candidate and valid node in this type because there is no downstream condition.

- **Type-B** - In  $C_c$ , all the upstream paths lie in both  $C_i$  and  $C_f$ .  $s$  can be updated if all downstream paths, after updating  $s$ , lie in either  $C_i$  or  $C_f$ .
- **Type-C Nodes** - Type B upstream condition was not satisfied. But in  $C_c$ , all the upstream paths lie in  $C_f$ .  $s$  can be updated if all downstream paths, after updating  $s$ , lie in  $C_f$ .
- **Type-D Nodes** - Neither Type A, Type B or Type C upstream condition was satisfied. In  $C_c$ , all upstream paths lie in  $C_i$ . In this case there need to be two downstream conditions. The first condition states that there is a path from  $s$  to  $H_2$  after the update. This is so that there are no sinks between  $s$  and  $H_2$ . This condition is required, because in this case, there are no upstream  $C_f$  paths. Since there is one sink and one source, having an upstream  $C_f$  path is enough to ensure that there will be a downstream  $C_f$  path after the update. The second condition states that all downstream paths, after updating  $s$ , must lie in  $C_i$ .
- **Type-E Nodes** - A node which does not satisfy any of the above upstream condition is a Type-E node. There are some upstream paths in  $C_i$  and some upstream paths in  $C_f$  but these two sets are disjoint. To update this node  $s$ , all downstream paths, after the update

should be in both  $C_i$  and  $C_f$ .

The upstream conditions in Figure 4.1 are exhaustive and mutually exclusive. And for each upstream condition, if the corresponding downstream condition is not satisfied, then updating the node will result in a inconsistent state. Hence, the stated conditions are exhaustive and any node that can be updated must satisfy one of the conditions in Figure 4.1.

The basic functionality of the ORDERUPDATE algorithm is searching for and updating valid nodes. We start with a set of nodes  $N$  to be updated, which initially contains all nodes, and after updating we remove the updated nodes from  $N$ . Finally,  $N = \emptyset$  and  $C_c = C_f$  since all nodes are updated.

## 4.2 Correctness and Completeness of ORDERUPDATE Algorithm

The upstream conditions in Figure 4.1 are exhaustive. The corresponding downstream conditions for each type need to be satisfied for updates to be consistent. Hence if Algorithm 1 produces a sequence, it is correct. Since the upstream conditions are exhaustive, if any node was not included in  $U$  in Line 14 of Algorithm 1, then it can not be updated in  $C_c$ .

**Property 1:** Removing any paths from  $C_c$  does not make  $C_c$  an inconsistent state. All remaining paths are still consistent. Another way of saying this is that removing some paths upstream or downstream to a valid node in  $C_c$  maintains validity of the node in  $C_c$ .

**Property 2:** Once a  $C_f$  path is established between two nodes, it can not be broken by any other update. This is because any update removes  $C_i$  paths from  $C_c$  but does not remove  $C_f$  paths from  $C_c$ .

**Lemma 1:** If  $T = UVnY$  is a valid sequence, then if  $n$  was valid after updating sequence  $U$ ,  $T' = UnV'Y$  is valid sequence where  $V'$  is a permutation of  $V$ , i.e  $V' = \pi(V)$ .

Updating  $n$  before any nodes in  $V$  adds some paths to  $C_c = upd(C_i, U)$  and removes some paths. From Property 1, we know that removing paths from  $C_c$  does not change the validity of nodes in  $V$ . But there are some nodes for which upstream or downstream paths were added. We now prove

that there exists an order  $V'$  in which we can update nodes in  $V$  to form an equivalent sequence  $T'$ .

Let us argue for all nodes  $f$  in  $V$  moving along the sequence  $V$  from left to right. Here  $C_c = \text{upd}(C_i, UV(f))$  where  $V(f)$  denotes the longest prefix of  $V$  that ends before  $f$  and  $C'_c = \text{upd}(C_i, UnV')$  where initially  $V'$  is empty sequence:

- Case 1 - There exists some upstream paths from  $n$  to  $f$  in  $C'_c$  which were not in  $C_c$ - Node  $f$  is downstream from  $n$ . Any upstream paths added to  $C'_c$  are from  $C_f$ . This is because updating the node  $n$  removes  $C_i$  edges and adds  $C_f$  edges to  $C'_c$ .  $f$  could be a valid node of one of the following types in  $C_c$ :

\* TA: This node was disconnected in  $T$ , so the only upstream paths in  $C'_c$  are from  $n$ . All paths from  $n$  to  $f$  in  $C'_c$  are in  $C_f$  but not  $C_i$ , so downstream paths from  $f$  to  $H_2$  are in  $C_f$ .  $f$  is not updated still all downstream paths in  $C'_c$  are in  $C_f$ . So outgoing edges from  $f$  in  $C_i$  are in  $C_f$ . We do not update  $f$  here. This is because all downstream paths from  $f$  in  $C'_c$  are already in  $C_f$ . Since all outgoing edges from  $f$  in  $C_i$  are in  $C_f$ , updating  $f$  would have only added some additional paths to  $C'_c$ . From Property 1, not updating  $f$  does not affect the validity of any other node in the sequence  $V$ . We refer to these nodes that we left out as **C1-TA** nodes and we will add them to  $V'$  in some other way.

\* TB: A Type-B node becomes a Type-C node if a upstream  $C_f$  path is added to it. Since there is an upstream  $C_f$  only path to  $f$  from  $n$ , all downstream paths from  $f$  in  $C'_c$  are in  $C_f$ . Paths starting from  $f$  in  $C'_c$  are a subset of the paths in  $C_c$  because some nodes in the sequence  $V$  were not updated. Paths in  $C_c$  after updating  $f$  were either in  $C_i$  or  $C_f$ . Paths which were in  $C_i$  must exist before updating  $f$  as well. If any of these  $C_i$  paths existed in  $C'_c$ , then they have to be in  $C_f$  as in  $C'_c$  all paths downstream from  $f$  are in  $C_f$ . Hence all paths after updating  $f$  will be in  $C_f$ . Update  $f$ .  $V' = V'f$ .



- \* TC: A Type-C node stays a Type-C node if some upstream paths from  $C_f$  are added to  $C'_c$ . This node can be still valid. We update it here.  $V' = V'f$ .
  - \* TD: Updating a Type-D node does not add any paths to its current configuration. It only removes some paths from it. Updating  $f$  may remove some paths from  $C'_c$  but from Property 1, this update is safe.  $V' = V'f$ .
  - \* TE: The Type-E downstream condition is very strong. Any node satisfying this condition can be updated no matter what type of candidate it is. So adding an upstream path may change the candidacy of this node but its downstream condition will still be fulfilled.  $f$  is still valid for update.  $V' = V'f$ .
- Case 2 - There exist some downstream paths from  $f$  to  $n$  in  $C'_c$  which were not in  $C_c$ . Node  $f$  is upstream from  $n$ . Any downstream path added was in  $C_f$  because updating  $n$  can only add  $C_f$  paths to  $C'_c$ . There can not be any  $C_i$  only paths from  $f$  to  $n$  in  $C'_c$  else there would be inconsistency on the downstream  $C_f$  paths. So, all paths from  $f$  to  $n$  in  $C'_c$  are in  $C_f$ . Since the paths added in  $C'_c$  are also  $C_f$  paths,  $f$  can be updated because updating  $f$  can only add more  $C_f$  paths between  $f$  and  $n$ .  $V' = V'f$ .
  - Case 3- There exist no downstream paths to or upstream paths from  $f$  in  $C'_c$  which were not in  $C_c$  - Since no paths were added,  $f$  stays valid and can be updated.  $V' = V'f$ .
  - Case 4- There exist both downstream paths to and upstream paths from  $f$  in  $C'_c$  which were not in  $C_c$  - This case is not possible since there can not be any cycles in  $C'_c$ .

So far, we removed C1-TA nodes from sequence  $V$  and created a subsequence  $V'$ . However we need to add these nodes to  $V'$  in some order to make sure that  $V' = \pi(V)$ . To achieve this we find the C1-TA node which has no other C1-TA node as its descendant in  $C_f$ . We update this node, add this node to  $V'$  and repeat this process until no C1-TA node is remaining. There is always one such node without C1-TA descendants in  $C_f$  because there are no cycles in the graph. We shall now prove that updating this node  $f$  maintains consistency. Let  $C_1 = \text{upd}(C_i, UVn)$  and  $C'_c = C_2 = \text{upd}(C_i, UnV')$ . The only difference between  $C_1$  and  $C_2$  is that some  $C_f$  paths in  $C_1$  are

missing from  $C_2$  because some C1-TA nodes were not updated. Updating  $f$  in  $C_2$  will add these paths to  $C_2$ . Since there is no C1-TA descendant, updating  $f$  will make  $C_1$  and  $C_2$  identical w.r.t paths starting at  $f$ .

We have obtained  $V' = \pi(V)$ . After this point, since updating the same set of nodes in any order leads to the same configuration,  $upd(C_i, UVn) = upd(C_i, UnV')$ , nodes in  $Y$  can be updated in sequence. Hence we proved that  $T' = UnV'Y$  is a correct sequence.

**Theorem** - Algorithm 1 generates a valid order of updates if there exists one.

Let  $Q = s_1, s_2, \dots, s_n$  be an valid sequence of updates, and  $Q_{alg} = s'_1, s'_2, \dots, s'_n$  be the solution generated by Algorithm 1. Let  $r$  be the first node s.t.  $\forall i < r : s_i = s'_i$ . Then using Lemma 1, there is another sequence  $Q' \equiv Q$  s.t.  $\forall i \leq r : s_i = s'_i$ . Using this argument for every index from  $i$  to  $n$ , we can find a valid sequence  $Q'' \equiv Q_{alg}$ .

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**Algorithm 1: ORDERUPDATE**


---

**Input:** Set of nodes to be updated  $N$ , Network Initial Configuration  $C_i$ , Network Final Configuration  $C_f$

**Result:** An order of consistent node updates

```

1  $W \leftarrow \emptyset$  // Waitlist is initially empty
2  $C_c \leftarrow C_i$  //  $C_c$  starts with the initial value of  $C_i$ 
3 while  $C_c \neq C_f$  // Stop when  $C_c$  and  $C_f$  are equal
4 do
5    $V_a \leftarrow CN_a \leftarrow \{s \mid s \in N \wedge \nexists p = path(H_1, s, C_c)\}$ 
   // Type-A Valid/Disconnected Nodes
6    $CN_b \leftarrow \{s \mid s \in N \wedge s \notin CN_a \wedge (\forall p \in paths(H_1, s, C_c) : (p \in C_i \wedge p \in C_f))\}$ 
   // Type-B Candidates
7    $V_b \leftarrow \{s \mid s \in CN_b \wedge (\forall p \in paths(s, H_2, upd(C_c, s)) : p \in C_i \vee p \in C_f)\}$ 
   // Type-B Valid Nodes
8    $CN_c \leftarrow \{s \mid s \in N \wedge s \notin CN_a \wedge s \notin CN_b \wedge (\forall p \in paths(H_1, s, C_c) : p \in C_f)\}$ 
   // Type-C Candidates
9    $V_c \leftarrow \{s \mid s \in CN_c \wedge (\forall p \in paths(s, H_2, upd(C_c, s)) : p \in C_f)\}$ 
   // Type-C Valid Nodes
10   $CN_d \leftarrow \{s \mid s \in N \wedge s \notin CN_a \wedge s \notin CN_b \wedge s \notin CN_c \wedge (\forall p \in paths(H_1, s, C_c) : p \in C_i)\}$ 
   // Type-D Candidates
11   $V_d \leftarrow \{s \mid s \in CN_d \wedge paths(s, H_2, upd(C_c, s)) \neq \phi \wedge (\forall p \in paths(s, H_2, upd(C_c, s)) : p \in C_i)\}$ 
   // Type-D Valid Nodes
12   $CN_e \leftarrow \{s \mid s \in N \wedge s \notin CN_a \wedge s \notin CN_b \wedge s \notin CN_c \wedge s \notin CN_d\}$ 
   // Type-E Candidates
13   $V_e \leftarrow \{s \mid s \in CN_e \wedge (\forall p \in paths(s, H_2, upd(C_c, s)) : p \in C_i \wedge p \in C_f)\}$ 
   // Type-E Valid Nodes
14   $U \leftarrow V_a \cup V_b \cup V_c \cup V_d \cup V_e$ 
15  if  $U = \emptyset$  then
16    | EXIT // No consistent order of updates exists
17  end
18   $s = PickAndWait()$  // By default, pick a random node, and wait
19   $C_c \leftarrow C_c - \{e \mid e = edge(s, t) \in C_i\}$  // Remove old outgoing edges from  $C_c$ 
20   $C_c \leftarrow C_c \cup \{e \mid e = edge(s, t) \in C_f\}$  // Add new outgoing edges to  $C_c$ 
21   $N \leftarrow N - \{s\}$  // Remove updated nodes from node list
22 end

```

---

## Chapter 5

### Minimizing Waits

In the previous section, we showed that Algorithm 1 produces a consistent order of updates. In this section, we shall extend Algorithm 1 by modifying the *PickAndWait()* subroutine on Line 18.

#### 5.1 Purpose of Waits

When Algorithm 1 picks a node to update, it removes all outgoing edges from it on Line 19, and updates the current network configuration. In the next iteration, it picks a node which is valid in the current configuration. If we did not flush the packets on these old edges, our network may not have actually reached the current configuration we assume it to be in. In Figure 2.3, we need to have a wait between B and D.

#### 5.2 Condition for Waits

We require a wait if there is an non-flushed  $C_i$ -only path upstream which got removed and there is a  $C_f$  only path in the current configuration after the update. These conditions are formalized in the *waitUp()* and *waitDown()* clauses in Algorithm 2, in Line 1 and Line 2. To keep track of flushed paths, we maintain a waitlist  $W$ . Any updated node which had traffic flowing through it on a  $C_i$ -only path would be added to waitlist  $W$ . We refer to these nodes as **waitlist** nodes. A node which has doesnot have any upstream  $C_i$  path is called a **non-waitlist** node. An example is this is C1-TA node from Lemma 1. Any node which has an ancestor in  $W$  on a  $C_i$ -only path which

was previously removed needs to wait.

### 5.3 A Greedy strategy

We define priorities and update nodes based on these priorities. The idea here is to delay waits. Delaying waits would allow the waitlist to build up as much as possible before it is flushed. We try to keep the wait as far down the sequence as possible. This presents the possibility that some of the waits in a non-optimal sequence would be pushed out of the sequence. Any node which does not need to wait is given a higher priority, priority  $P_0$ . Nodes that need to wait are given priority  $P_1$ .

### 5.4 Proof of Optimality

**Property 3:** If a node  $n \in P_0$  in configuration  $C_1 = upd(C_i, S)$ , where  $S$  is a sequence of nodes, then for all sequences  $S'$  of nodes not in  $S$ , if  $n$  is valid in configuration  $C_2 = upd(C_i, SS')$ ,  $n \in P_0$  in  $C_2$ .

Suppose in  $C_2$ ,  $n \in P_1$ , then either an upstream condition was added to  $n$  in  $C_2$  or a downstream condition was added to  $n$  in  $C_2$  or both.

- Case 1- Upstream condition was added in  $C_2$  - This means that in  $C_1$ ,  $n$  had an upstream  $C_i$  path which got removed in  $C_2$ . There can't be a downstream  $C_f$ -only path in  $upd(C_1, n)$ . So, If an upstream condition was added, downstream condition was added too. This case is equivalent to case 2.
- Case 2- Downstream condition was added in  $C_2$  - The downstream  $C_f$ -only path was not present in  $upd(C_1, n)$  but is present in  $upd(C_2, n)$ . This path is added by some node  $r$  in sequence  $S'$ .  $r$  was downstream from  $n$  and connected to it in  $C_1$ . In  $upd(C_1, n)$ ,  $n$  had no downstream  $C_f$ -only paths, so  $n$  and  $r$  are connected by a  $C_i$ -only path. So ancestors of  $n$  are ancestors of  $r$  and would have been flushed when  $r$  was updated. Also, since  $r$  added a  $C_f$ -only path downstream, all  $C_i$ -only paths upstream have been removed. In this case,

$$n \in P_0.$$

- Case 3- Both upstream and downstream condition was added in  $C_2$ . This case is already covered by case 2.

This proves Property 3. Any node that becomes priority  $P_0$  at some time, it stays priority  $P_0$  whenever it is valid in future.

**Lemma 2:** If  $T = UVnY$  is a valid sequence, and after updating sequence  $U$ ,  $n \in P_0$ , then  $T' = UnV'Y$  is valid sequence with lesser or equal waits. Here  $V'$  is a permutation of  $V$ , i.e  $V' = \pi(V)$ .

This lemma is an extension of Lemma 1. In addition to  $n$  being a valid node,  $n$  is a Priority  $P_0$  node. We shall prove that if  $n$  satisfies these constraints, then  $T'$  can be constructed using the same  $V'$  as in Lemma 1. In  $T'$ , after updating  $U$  and  $n$ , we first update all nodes in  $V$  which were not C1-TA, in order. We then update all C1-TA nodes in a downstream first order. This way, we build  $V'$  from  $V$  in two phases:

- Phase 1 - In this phase we update all but C1-TA nodes. Let us argue for each  $f$  in Phase 1. For the following proof,  $C_c$  refers to the configuration before  $f$  is updated in  $T$  and  $C'_c$  is the corresponding configuration in  $T'$ :

- \* Case 1 - If  $f \in P_1$  in  $T$ , then in  $T'$ ,  $f \in P_0$  or  $f \in P_1$ . In either case,  $f$  does not add any waits in  $T'$  as compared to  $T$ .

- \* Case 2 - If  $f \in P_0$  in  $T$ . If  $f \in P_0$  in  $T'$ , then no additional waits are added to  $T'$  due to  $f$ . However, if  $f \in P_1$  in  $T'$ ,  $waitUp(f)$  and  $waitDown(f)$  are both true in  $T'$ .

This change in priority could be due to one of the following reasons:

- Upstream condition was added in  $T'$ . Downstream condition was unchanged.

The change in upstream condition could be because of two reasons:

- (1) Some nodes, which were flushed from the waitlist in  $T$ , were not flushed in  $T'$ . This was because:

- (a) A C1-TA node which had priority  $P_1$  in  $T$  was not updated. We add a wait a before  $f$ . This wait was shifted from one node to another. Using property 3, we can say that the C1-TA node would have priority  $P_0$  whenever it is updated. This wait is possibly delayed with respect to Phase 1 nodes. Phase 2 nodes do not get added to the waitlist and so, only considering the nodes that will get added to waitlist, this wait either stays in its relative position or moves to the right.
  - (b) Some node before  $f$  satisfied Case 1 and did not wait. We wait before  $f$ . Here again, we shift the wait from one node to another. The wait here has shifted right.
- (2) Some upstream  $C_i$ -only upstream paths were not removed from  $C_c$  are removed in  $C'_c$ . Since the downstream condition is unchanged, it exists in  $C_c$ , and  $f$  cannot have any upstream  $C_i$ -only paths. All such paths would have been removed from  $C_c$  as well. This case is not possible.
- Downstream condition was added in  $T'$ . The upstream condition may or may not have been added in  $T'$ . A downstream  $C_f$ -only path exists in  $T'$  which did not exist in  $T$ . We saw in Lemma 1, that not updating C1-TA nodes does not add paths in the network. This path could only have been added because of  $n$ , and that too, only if  $n$  is downstream from  $f$  in  $C_c$  and  $C'_c$ . There is a path from  $f$  to  $n$  in  $C_c$ . This path can not be  $C_f$ -only since  $f$  did not satisfy the downstream condition in  $T$ . This path is a  $C_i$  path. Since a downstream condition for  $f$  can only be caused by  $n$ , this path exists in  $C'_c$  as well. When  $n$  was updated, it added a  $C_f$ -only downstream path, so all upstream  $C_i$ -only paths would have been removed. Additionally, since  $n$  was updated in  $T'$ , we know that all ancestors on these  $C_i$ -only upstream paths were already flushed or non-waitlist nodes. Since all ancestors of  $f$  were flushed or non-waitlist nodes, it

is not possible for  $f$  to have priority  $P_1$ .

- Phase 2 - These nodes do not have upstream  $C_i$  paths so they do not get added to the waitlist. Additionally, there does not need to be more than one wait in this phase. This is because, before the beginning of this phase, all upstream  $C_i$ -only paths would have been removed (these nodes were disconnected in  $C_c$ ), and ancestors along these paths, added to waitlist. Adding one wait would flush all these ancestors at once.

\* If  $f$  had priority  $P_1$  in  $T$ , then if it still has priority  $P_1$ , then the wait before it was not shifted and if we wait before  $f$  there would not be any additional waits in  $T'$  as compared with  $T$ .

\* If  $f$  had priority  $P_0$  in  $T$ , and  $P_1$  in  $T'$ , there can be 2 reasons for this, like the corresponding case in Phase 1:

- Upstream condition was added. Downstream condition was unchanged. Since these nodes have all their  $C_i$ -only upstream paths removed from  $C'_c$ , the only way an upstream condition was added was if some nodes were not flushed. In Phase 1, all waits either stayed in place or were shifted right. The only way that some nodes that were flushed in  $T$  were not flushed in  $T'$  is that one wait shifted right and got dropped. We can add this wait here.
- Downstream condition was added. This case is the same as the case for Phase 1 and is not possible.

We proved that waits in sequence  $UnV' \leq$  waits in sequence  $UVn$ . We have also seen that all waits are either at the same position or are delayed. This delay in waits would mean that at the end of  $UnV'$ , more nodes would be flushed and fewer nodes would be on the waitlist as compared to  $UVn$ . So, waitlist in sequence  $UnV' \subseteq$  waitlist in sequence  $UVn$ . This would mean that while updating  $Y$  in  $T'$ , the number of waits either stays the same or reduces. Hence, we proved That waits in sequence  $T' \leq$  waits in sequence  $T$ .



**Lemma 3:** If  $T = UVnY$  is a valid sequence, and after updating sequence  $U$ ,  $P_0 = \emptyset \wedge n \in P_1$ , then  $T' = UnV'Y$  is valid sequence with lesser or equal waits. Here  $V'$  is a permutation of  $V$ , i.e  $V' = \pi(V)$ .

Here again,  $V'$  is formed from  $V$  in the same way as before. In  $T$ , there is a wait before any node in  $V$  is updated because  $P_0 = \emptyset$  after updating sequence  $U$ . If  $n$  is updated before  $V$ , then by Property 3, the first node in  $V'$  would have to be in  $P_0$ , and thus one wait from  $V'$  is removed and added before  $n$  in  $T'$ . The argument for rest of the nodes in  $V'$  and  $Y$  would stay the same. In  $T$  if there was a wait before  $n$ , then there still is one. However, if in  $T$ , there was no wait before  $n$ , then one wait is borrowed from nodes in  $V'$ . Overall, waits in sequence  $T' = UnV'Y \leq$  waits in sequence  $T = UVnY$ .

**Theorem 2:** Algorithm 1 and Algorithm 2 produce a valid order of updates with minimal number of waits if there exists one.

Let  $Q = s_1, s_2, \dots, s_n$  be an optimal valid sequence, and  $Q_{alg} = s'_1, s'_2, \dots, s'_n$  be the sequence generated by Algorithm 1 and Algorithm 2. Let  $r$  be the first node s.t.  $\forall i < r : s_i = s'_i$ . If  $s'_r \in P_0$ , then by Lemma 2, we can generate a sequence  $Q' \equiv Q$  s.t.  $\forall i \leq r : s_i = s'_i$ . If  $s'_r \in P_1$ , then by Lemma 3, we can again generate a sequence  $Q' \equiv Q$  s.t.  $\forall i \leq r : s_i = s'_i$ . Using this argument for every index from  $i$  to  $n$ , we can find a valid sequence  $Q'' \equiv Q_{alg}$ .

---

**Algorithm 2:** PickAndWait
 

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**Result:** Return a node that minimizes waits in the sequence

```

1  $waitDown(q) = (\exists p \in paths(q, H_2, upd(C_c, q)) : p \in C_f \wedge p \notin C_i)$ 
2  $waitUp(q) = (\exists p \in paths(H_1, q, C_i) : p \notin C_c \wedge p \notin C_f \wedge (\exists s \in nodes(p) : s \in W))$ 
3  $P_0 \leftarrow \{s \mid s \in U \wedge \neg(waitDown(s) \wedge waitUp(s))\}$ 
   // Nodes we can update without waiting
4  $P_1 \leftarrow \{s \mid s \in U \wedge waitDown(s) \wedge waitUp(s)\}$ 
   // Nodes we can not update without waiting first
5 if  $P_0 \neq \emptyset$  then
6   |  $R = random(P_0)$  // Return any node in  $P_0$ 
7 end
8 else
9   | WAIT // Need to wait before updating  $P_1$  nodes
10  |  $R = random(P_1)$  // Return any node in  $P_1$ 
11 end
12 if  $\exists p \in paths(H_1, R, C_c) : p \in C_i$  // If  $R$  is a waitlist node, add it to  $W$ 
13 then
14   |  $W \leftarrow W \cup \{R\}$ 
   //  $R$  is a waitlist node if it has traffic incoming on a  $C_i$  path
15 end
16 return  $R$ 

```

---

## Chapter 6

### Related Work

There is a considerable amount of work related to avoiding erratic transient behavior that arises while updating routes in a Software Defined Network.

**Consistency.** Our work is motivated by earlier work on network updates in SDN [9] that proposed the notion of per-packet consistency and provided mechanisms for consistent updates like two phase updates.

**Exponential Search Based Algorithms.** Dionysus [4] achieves fast, consistent network updates through dynamic scheduling of rule updates. Swan and zUpdate add support for bandwidth guarantees [3, 5]. The CCG [11] framework supports customizable consistency policies during network updates. McClurg et al. [8] present a update synthesis algorithm based on counter-example guided search and incremental model checking. More recent work [7] introduces event-driven consistent updates using network event structures to model constraints on updates. The FLIP [10] Algorithm computes policy preserving per-flow sequences.

**Complexity results.** Ludwig et al. [6] study schedules that minimize controller interactions (rounds) in a loop-free manner and show that deciding whether a  $k$ -round schedule exists, is NP-complete for  $k = 3$ . Förster et al. [2] show that for the basic consistency properties of loop and blackhole freedom, fast updates are NP-hard optimization problems and present an algorithm with provably minimal dependency structure. The constraint of per-packet consistency in our problem includes loop freedom and blackhole freedom, and finds a sequential update schedule. Brandt et al. [1] give a polynomial time algorithm to decide if congestion free configuration change is possible when

flows are splittable. This notion of congestion freedom is different than our notion of per-packet consistency as packets may take a path which lies partially in the old configuration and partially in the new configuration.

## Chapter 7

### Conclusion

Our discussion so far can be summarized into the following high level points:

- We presented a polynomial time algorithm, OrderUpdate, to find an order of consistent updates for the nodes in a network with unweighted directed edges and single packet type.
- We proved that the OrderUpdate algorithm is correct and complete.
- We then presented a modification, PickAndWait, to the OrderUpdate algorithm which finds an order of consistent updates with minimal number of waits.
- We proved that the OrderUpdate algorithm, with the PickAndWait modification, is correct, complete and optimal.

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