Spring 2013

Measuring and Modeling Radiation Loss in Superconducting Microwave Re-entrant Cavities

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Measuring and Modeling Radiation Loss in Superconducting Microwave Re-entrant Cavities

by

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A thesis submitted to the Faculty of the University of Colorado in partial fulfillment of the requirements for graduation with Honors in the degree of Bachelor of Science in Engineering Physics

Department of Physics

2013

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This thesis describes the potential use of re-entrant microwave resonant three-dimensional cavities as low-loss electrical component in strongly coupled electromechanical systems. In particular, I have investigated radiative processes as a loss mechanism of a re-entrant cavity in a geometry similar to those used in electromechanical experiments undertaken by the Lehnert lab. First, I present the verification of a lumped-element model describing the cavity resonance, a key electrical characteristic of a microwave re-entrant cavity, and its dependence on dimensions critical to strong electromechanical coupling. Next, I focus on modeling and measuring the total power radiating from a re-entrant cavity. To this end, I evaluated the effectiveness of surrounding the re-entrant cavity with a waveguide “choke” flange as a method of reducing total radiated power. The experimental results show the energy dissipation rate from radiation can be reduced by at least $\sim 5 \times 10^4$ after incorporating a radiation choke flange. This experiment aimed to inform whether a cavity with a quality factor that is limited by radiation losses is satisfactory to achieve single-photon coupling $g_0$ greater than $\kappa$, the electromagnetic energy dissipation rate, in an electromechanical device. The pursuit of strong single-photon electromechanical coupling is the motivation of my investigations of radiation loss in superconducting microwave re-entrant cavities.
Acknowledgements

I would like to thank Konrad Lehnert, Adam Reed, Reed Andrews, Joe Kerckhoff, Tauno Palomaki, Jen Harlow, Will Kindel, Mike Schroer, Hsiang-Ku, Mehmet Anil, Gerwin Koolstra, and the JILA instrument shop personnel for aiding my growth as a scientist. I could not have been more fortunate to be surrounded by such a supportive, intelligent research team.
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Chapter 1

Microwave Optomechanics

1.1 Motivation

The field of optomechanics studies systems that couple electromagnetic signals—light or electricity—with mechanical elements such as membranes, cantilevers, and other structures. These devices enable studies of macroscopic objects that behave quantum mechanically. Mechanically compliant structures can also interact with all fundamental forces of nature; this makes them useful as an intermediary between electrically inactive systems (e.g. gravitational waves) and optical/electrical measurement tools.

An example application of an electromechanical system for studying macroscopic quantum objects is the cooling of a mechanical element to its ground state [5]. An effective way of manipulating and measuring a mechanical oscillator is to transduce the mechanical information into electromagnetic fields, which is the choice medium for measurement tools. Furthermore, an example of a mechanical oscillator that serves as an intermediary for precise measurement of an electrically inactive system is magnetic resonance force microscopy (MRFM). MRFM uses a small cantilever to measure the magnetic dipole force between the nuclear spins in the sample and a nearly probe particle; this cantilever is then coupled to a laser interferometer for measurement. The use of mechanical elements as an intermediary coupling device improves resolution of spatial measurements beyond that of purely optical/electrical measurement techniques.

In addition, precise state-manipulation of the mechanical element using electromagnetic waves enables the storage of a quantum state in one medium while measuring with another. A few
examples of the utility in transferring quantum states between a mechanical and an electrical element are given as follows. First, one medium can be better-suited to a given task over the other. For example, mechanical oscillators can have very long decoherence times; due to their low loss rates, a quantum state can be stored in a mechanical element for minutes or more [3]. Electromagnetic waves, on the other hand, are well-suited for transmitting quantum states over long distances. Coupling these media in a low-loss fashion makes strides toward a flexible quantum information processing network. Microwave optomechanics in particular has shown the capability to transfer and readout coherent quantum mechanical states between microwave fields and a mechanical oscillator [16]; this is particularly crucial in realizing quantum memories.

My contribution to the field of microwave optomechanics is motivated by the pursuit to achieve strong single-photon electromechanical coupling. Strong single-photon electromechanical coupling means that the rate at which a single quanta of energy is transferred between the electromagnetic field and the mechanical oscillator is larger than the rate at which each element dissipates an energy quanta to their respective environments. For electromagnetic fields, the energy quanta is the photon, while the energy quantum in a mechanical quantum harmonic oscillator is the phonon. A figure of merit that encompasses these loss rates is the cooperativity, where

\[ C \equiv \frac{g_0^2}{\kappa \Gamma_m} \]  

In equation 1.1.1, \( g_0 \) is the single-photon coupling rate, \( \kappa \) is the energy dissipation rate of the electrical oscillator (referred to as a cavity) and \( \Gamma_m \) is the energy dissipation rate of the mechanical element. See Figure 1.1 for a diagram of the energy transfer rates for an electromechanical system. To achieve strong electromechanical coupling, then, we desire \( g_0 > \max[\kappa, \Gamma_m] \). Note that strong field-enhanced coupling, \( g_0 \sqrt{N} > \max[\kappa, \Gamma_m] \), where \( N \) is the number of photons in the electromagnetic field, has been achieved previously (Teufel 2011 [18]). Achieving \( g_0 > \max[\kappa, \Gamma_m] \) would be a powerful result in the field of optomechanics, enabling interesting studies such as the preparation of non-gaussian quantum mechanical oscillator states in the steady state [13].
Figure 1.1: Diagram representing the relative coupling rates between the measurement (left), the electrical cavity (middle) and the mechanical oscillator (right). The electromechanical system is immersed in a cryogenic thermal environment, and the two oscillators are ideally the same temperature. The loss rates to the environment are $\kappa_{\text{int}}$ and $\Gamma_m$ for the electrical and mechanical oscillators, respectively. The microwave “cavity” couples to the measurement probe at coupling rate $\kappa_{\text{ext}}$. Strong coupling requires $g_0 \gtrsim (\kappa_{\text{int}} + \kappa_{\text{ext}})$. 
next section, I will provide some physical intuition for the single-photon coupling rate $g_0$ for an example electromechanical system.

1.2 Physical Background

An effort in microwave electromechanics is to explore mechanically compliant objects in the quantum regime. Observing quantum behavior in a micron-scale mechanically compliant device generally means the motion of the mechanical element is flexural, as opposed to bulk acoustic waves or the spinning/sliding that is common mechanical motion at larger scales. When the mechanical element is a component of an electrical microwave circuit, the equation of motion of the mechanical device and the electrical circuit is parametrically coupled. This means that the motion of the mechanical device changes a parameter of the electrical device—often, this is the resonant frequency of an LC circuit. An example of such a device is given in Figure 1.2. The equations of motion for the system can be solved using the Lagrangian formalism. For example, the system in Figure 1.2 would have a Lagrangian [8]:

$$L = \frac{1}{2} \left( Lq^2 + m\dot{x}^2 - \frac{q^2}{C(x)} - k_x x^2 \right) + qV + F_{ext}x$$  \hspace{1cm} (1.2.1)$$

One may account for the dissipation by adding the dissipation from the resistor $R$ and the dissipation to any resistance in the signal impedance $Z_0$ to the equations of motion derived from the Lagrangian. This method is easily extended to the quantum analog using the Hamiltonian formulation; however, dissipation is not so trivial to include in the quantum description of an electromechanical circuit.

The quantum Hamiltonian for the system is

$$\hat{H} = \hbar \omega_c (\hat{a}^\dagger \hat{a} + \frac{1}{2}) + \hbar \Omega_m (\hat{b}^\dagger \hat{b} + \frac{1}{2}) + \hat{H}_I$$  \hspace{1cm} (1.2.2)$$

where $\hat{a} (\hat{a}^\dagger)$ are the annihilation (creation) operators of photons in the microwave field and $\hat{b} (\hat{b}^\dagger)$ are the annihilation (creation) operators of phonons in the oscillator, respectively. Thus, the first term is the energy of the microwave field, the second term is the energy of the mechanical oscillator,
Figure 1.2: Example of a parametrically coupled electromechanical system. The dashed box surrounds what is considered the electromechanical device, with the signal source outside. The signal source has an internal impedance $Z_0$ and is driving the circuit with amplitude $V_0$. The capacitor in the LRC circuit is modified by the motion of the mechanical oscillator, shown here as a spring-mass system, where one capacitor plate is the mass. \[8\].
and $\hat{H}_I$ is the interaction Hamiltonian. The interaction Hamiltonian is formed by the parametric coupling of the mechanical oscillator and the electrical cavity. Defining $\hat{x}$ as shown in Figure 1.2 and $d$ as the equilibrium distance between the capacitor plates, I will present a calculation of $g_0$ in the limit $\hat{x} \ll d$. This is the weak parametric coupling limit, and so a linear approximation in $\hat{x}$ of the cavity frequency $\omega_c(x)$ is sufficient:

$$\omega_c(x) \approx \omega_c + \left(\frac{\partial \omega_c}{\partial x}\right) \hat{x}$$

$$\omega_c(x) \approx \omega_c - G \hat{x}$$

(1.2.3)

I’ve defined $G = -\partial \omega_c / \partial x$ as the cavity frequency shift per motion. Rewriting $\hat{x}$ as $\hat{x} = x_{zp} \left(\hat{b}^\dagger + \hat{b}\right)$, where $x_{zp}$ is the zero-point motion of the oscillator, the interaction Hamiltonian becomes

$$\hat{H}_I = -\hbar g_0 \hat{a}^\dagger \hat{a} \left(\hat{b}^\dagger + \hat{b}\right)$$

(1.2.4)

In equation 1.2.4, $g_0$ is the linearized single-photon electromechanical coupling rate, defined as:

$$g_0 = G x_{zp}$$

(1.2.5)

For the coupling mechanism shown in Figure 1.2, $G$ can be written as:

$$G = -\omega_c \frac{1}{2C} \frac{\partial C}{\partial x}$$

(1.2.6)

As an example calculation, consider the parametrically coupled LRC circuit in figure 1.2. In the case of a vacuum-gap parallel-plate capacitor, the capacitance $C(x)$ in the linearized regime is

$$C(x) \approx \varepsilon_0 A \frac{\hat{x}}{d} \left(1 - \frac{\hat{x}}{d}\right)$$

$$\approx C_0 \left(1 - \frac{\hat{x}}{d}\right)$$

(1.2.7)

where $C_0$ is the equilibrium capacitance. Then, the change in capacitance per motion is

$$\frac{\partial C}{\partial x} = -\frac{C_0}{d}$$

(1.2.8)
Substituting equations 1.2.7 and 1.2.8 in to 1.2.6, we have

\[ G = \frac{\omega_c}{2C_0 (1 - \frac{\hat{x}}{d})} \approx \frac{\omega_c}{2d (1 + \frac{\hat{x}}{d})} \approx \frac{\omega_c}{2d} \]  

(1.2.9)

where I discarded the term that was second order in \( d \) since \( \hat{x}/d \ll 1 \). This means \( g_0 \) is

\[ g_0 = \frac{\omega_c x_{zp}}{2d} \]  

(1.2.10)

Applying experimentally reasonable values—say, \( x_{zp} \sim 0.5 \text{ fm}, \omega_c/2\pi \sim 7 \text{ GHz} \), and \( d \sim 100 \text{ nm} \)—to equation 1.2.10 gives \( g_0/2\pi = 18 \text{ Hz} \). It is often the case experimentally that \( \Gamma_m \ll \kappa \), so strong coupling is achieved when \( g_0 \gtrsim \kappa \). Thus, \( \kappa/2\pi \lesssim 18 \text{ Hz} \), corresponding to a \( Q_0 \gtrsim 3 \times 10^8 \) so to get single-photon strong electromechanical coupling.

The collective efforts of this project involve evaluating what an achievable \( g_0 \) could be for the re-entrant cavity geometry, as well as what loss rate \( \kappa \) the re-entrant cavity can reach. My thesis in particular aims to quantify the energy dissipation rate due to radiative processes in the re-entrant microwave cavity geometry. Ultimately, this research seeks to answer the question of whether or not this type of microwave cavity can reach \( g_0/\kappa \) above unity.
Chapter 2

The Re-entrant Cavity: A 3-D Distributed Electrical Circuit

2.1 The Geometry

Optoemechanical experiments typically involve an on-chip resonant microwave circuit as the electrical element, such as a coplanar waveguide or a microstrip resonator. An example shown in Figure 2.1. One drawback is that on-chip electromechanical systems require the mechanical element fabrication to be made during the electrical circuit fabrication. This limits material choices of the mechanical oscillator and the electrical circuit. A three-dimensional resonator geometry allows for separate fabrication of the electrical and the mechanical elements. We can engineer high-Q mechanical oscillators and high-Q electrical cavities in parallel and put them together in a modular fashion, enabling optimization of each independently.

Another drawback of this on-chip geometry is that there are a multitude of material boundaries present. There are metal-to-substrate, metal-to-air, and air-to-substrate interfaces, and at each of these boundaries power is dissipated due to absorption from amorphous dielectric materials that form at these interfaces. Though on-chip resonant circuits can address the problem of lossy dielectric materials, such as the use of vacuum-gap capacitors [2], these devices are still limited to an unloaded quality factor $Q_0 \sim 3 \times 10^4$. The linewidth of these devices are limited by lossy dielectric at the metal-to-air interface. To achieve strong electromechanical coupling, we seek an electrical resonator with a $Q_0 \sim 10^7$. Using a 3-D resonator enables us to address the dielectric loss on the surfaces of the conductor without disturbing the mechanical oscillator.

This reasoning motivates the transition to three-dimensional cavities, but we also need a
Figure 2.1: a) Photograph of a lumped-element microwave LC circuit with a spiral inductor and a mechanically compliant vacuum-gap capacitor b) Close view of the suspended membrane capacitor. The arrows point to the vacuum gaps and a crossover of the inductor. Figure used with permission (Teufel 2011 [18])

geometry that allows for strong coupling between the cavity and the mechanical oscillator. Recall that strong coupling is realized when the zero point motion of the oscillator shifts the cavity resonance by its linewidth or more ($g_0/\kappa \gtrsim 1$). To outline how electromechanical coupling is determined with 3-D cavities, I'll first consider a box-shaped cavity. The largest wavelength that would satisfy the boundary conditions of a box of length $l$ is $\lambda \approx 2l$. Therefore, the fundamental frequency of this cavity is $\omega \approx \pi c/l$. The frequency shift per change in the box dimension $l$, then, is $G \equiv \partial \omega/\partial l \approx -\omega/l$. Knowing $l \approx \pi c/\omega$, we see $G \approx -\omega^2/\pi c$. Because $G$ is quadratic in frequency, it appears ridiculous to choose a low-frequency electrical resonator, such as a microwave system, over one at high frequencies, such as an optical cavity.

The description above assumed a convex electromagnetic cavity. This means the electric and magnetic field energies are stored in the same region of the cavity volume. However, if we consider a geometry that is concave (i.e. we cannot draw a line from every point on the outer cavity edge to every other without penetrating the metal at least once), then the electric and magnetic field energies are stored in separate regions of the cavity. Thus, the spectral distribution can be made quite low relative to the cavity dimensions. The extreme limit is seen in lumped
LC circuits, which can have resonant wavelengths orders of magnitude longer than their largest dimension. In addition, the electric field localizes between the closely-spaced conductors, and so the field distribution becomes more sensitive to changes in that certain dimension. In turn, the frequency shift per change in length is of the form $G \propto \omega/d$, where $d$ is the gap spacing between the two walls. The dimension $d$ can be made very small relative to the other cavity dimensions, meaning for a specified frequency, $G$ for a concave cavity can be made much larger than $G$ for a convex cavity.

The appeal of microwave frequencies in optomechanics is largely because we seek a mechanical oscillator that can store quantum states for an extended period of time. If the mechanical oscillator is in a thermal state (i.e. when the oscillator energy $\hbar \omega$ is smaller than the thermal energy $k_B T$), the mechanical oscillator will be excited in a random, incoherent fashion. When $\hbar \omega/k_B T > 1$, the quantum decoherence rate of the mechanical oscillator is $\Gamma_m (k_B T/\hbar \omega)$. Therefore, to store pure quantum-mechanical states in the mechanical oscillator, we need the mechanical oscillator to be low-temperature. Experimentally, it is challenging to implement a high-finesse optical cavity in a cryogenic environment. A more convenient way to couple electromagnetic fields and a low-temperature mechanical element is to use electrical circuits. Microwave frequencies are appealing because, unlike the optical domain, they can be implemented in cryogenic environments using standard electronic components. In addition, microwaves are high-enough frequency such that quantum states can be realized in a commercially available cryostat. For example, $T = \hbar \omega/k_B$ gives a temperature $T = 240$ mK for a 5 GHz resonant circuit; temperatures an order of magnitude lower than this are commonly achieved in the laboratory.

Three-dimensional cavities are appealing because they offer a fabrication process that is separate from the fabrication of the mechanical element. The natural intersection of these considerations is to engineer a convex microwave cavity. This can be used in a cryogenic environment and can have very small dimensions when compared to a concave cavity in the same frequency range. The re-entrant cavity geometry is a promising concave microwave cavity for achieving strong electromagnetic coupling. It consists of an annular hole bored into metal, leaving a post in the center and
a gap between this post and the upper cavity wall. See Figure 2.2 for an example re-entrant cavity.

Intuition about the re-entrant cavity capabilities for strong coupling can be thought of as follows: buildup of charge between the post and the upper wall would produce a large voltage; hence, most of the electrical energy stored in the cavity is stored between the post and the upper wall. This region is akin to a parallel-plate capacitor, whose capacitance is $C \propto 1/d$, where $d$ is the gap between the plates. Therefore, the motion of a mechanically compliant upper wall would result in a capacitance that sensitively depends on this motion; hence, the cavity resonance is sensitive to the mechanical motion in the system. The re-entrant cavity is promising for both a small linewidth $\kappa$ and strong electromechanical coupling. Unloaded quality factors on order $10^5$ have been achieved [11], and they have been used as sensitive electromechanical transducers in experiments designed to detect gravitational waves [9].

2.2 Modeling Circuit Parameters

Because of its concave nature, re-entrant cavities can be modeled using a lumped-element approximation. When the gap between the center post and the upper cavity wall becomes very small, the electric field localizes in this region. In addition, the currents that flow down the center post, along the bottom, up the outer wall, and toward the center of the upper wall generate magnetic fields in the same direction in the region between the post and the outer wall (see Figure 2.2). Currents flowing along the upper wall toward its center generate magnetic fields that cancel with one another because of the radial symmetry. Thus, the magnetic fields are stored in the region between $r_0$ and $r_1$ below $(h - d)$. This understanding can pave the way for a lumped-element model of the re-entrant cavity. This enables a simple approach to cavity design by eliminating the necessity of a full, three-dimensional field solution.

Fujisawa [4] presented a model for several variants of re-entrant cavity. The model below applies to the geometry of re-entrant cavity specified in Figure 2.2. The inductance $L$ is defined
Figure 2.2: (a) Cross Section of a Re-entrant Cavity with a snapshot in time of the fundamental mode field distribution, where $E$ is the electric field, $B$ is the magnetic field, and $K$ is the current density that runs on the conducting surfaces. The dimensions outlined are referred to in the lumped element circuit model. (b) Re-entrant Cavity made in the Lehnert Lab. The hole bored in the bottom is used to couple the 50 Ω UT-85 semi-rigid coaxial cable that sends microwaves into the cavity. The nominal dimensions for this cavity are $r_0 = 4$ mm, $d = 0.5$ mm, $r_1 = 9$ mm, and $h = 2.5$ mm.
as
\[
\frac{L}{\mu_0} = \frac{h}{2\pi} \ln \frac{r_1}{r_0} \tag{2.2.1}
\]
while the capacitance includes a parallel-plate approximation plus a parasitic capacitive term:

\[
C = C_0 + C_1
\]

\[
\frac{C_0}{\epsilon_0} = \frac{\pi r_0^2}{d} \tag{2.2.2}
\]

\[
\frac{C_1}{\epsilon_0} = 4r_0 \ln \frac{e l_M}{d} \tag{2.2.3}
\]

where
\[
l_M = \sqrt{(r_1 - r_0)^2 + h^2} \tag{2.2.4}
\]
The formula for the resonant frequency \(f_0\) is the same as an LRC circuit:

\[
f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{2.2.5}
\]
The critical re-entrant cavity dimension that will govern the electromechanical coupling is the gap spacing between the mechanical membrane and the center post. Understanding the dependence of the resonant frequency of the cavity on this gap distance is imperative to designing effective electromechanical systems. To compare with the lumped-element model, I simulated the resonant frequency of a re-entrant cavity using a finite-element numerical software (Ansys HFSS) while varying the center gap height. The results are shown in Figure 2.3

### 2.3 Verifying the Lumped-Element Model

With theoretical and numerical predictions of cavity resonance \(f_0\) as a function of center-post gap distance \(d\), I next designed a way to test the circuit model experimentally. I built an apparatus that changed the gap distance \(d\) of a re-entrant cavity of the type and dimensions in Figure 2.2. This was done by depressing an aluminum diaphragm placed on top of the cavity, which changed the gap distance between the post and the upper wall. Mounted to the top of the cavity was an apparatus that held a 100 threads-per-inch adjuster screw in a bushing. The mount is pictured in Figure 2.4.
Figure 2.3: Theoretical description of resonant frequency versus gap distance $d$. 

Fujisawa Model Vs. HFSS Predictions 

$\begin{align*}
\text{Lumped-Element Model} \\
\text{HFSS Simulation}
\end{align*}$
Figure 2.4: (a) re-entrant Cavity with a flexible diaphragm serving as the upper cavity wall. The cavity was measured in transmission, as shown by the two ports extruding from the cavity bottom. (b) The device setup taken apart. On the left: the re-entrant cavity, with the coupling SMA launchers below. In the middle: the aluminum diaphragm, and below the plunger that was placed between the screw and the diaphragm. On the right: the diaphragm mount, with the fine-adjustment screw threaded into the brass bushing.
This screw pushed on the 0.016 " thick diaphragm with a “plunger” that was as wide as the center post between the screw and the diagram. This was done to keep the diaphragm flat in the region above the center post, as the screw diameter was smaller than the center post diameter. The cavity was outfitted with two coupling ports and measured in transmission, as measurement in reflection would have proved difficult due to the decrease in electric coupling rate as the center-post gap is made smaller. The squared magnitude transmission spectrum of port 2 from port 1 $|S_{21}|^2 = |V_2|^2 / |V_1|^2$ is presented in Figure 2.5, with each curve corresponding to a different center-post gap spacing.

In order to quantitatively compare this data to the predictions, I fit the data to the theory that was outlined in the preceding section with two free parameters: the unflexed gap distance $d$ and an error in height $h$. This is necessary because the unflexed diaphragm was depressed somewhat in the center prior to the experiment; therefore, it is nontrivial to determine the gap distance between the center post and the diaphragm by direct measurement. Fitting the measured resonant frequency versus the change in gap distance to the circuit model with two parameters $d$ and $h$ yielded $d = 0.299$ mm, meaning that the diaphragm was dented $\sim 0.2$ mm in the center (see Figure 2.4), and $h_{\text{fit}} = 0.9h_{\text{meas}}$ ($h_{\text{meas}} = 2.5$ mm). It makes sense that the average cavity height would be approximately 10% smaller than the unflexed height because I was depressing the upper cavity wall as I turned the adjustor screw. For these dimensions, the mean cavity height between $d = 0.5$ mm and $d = 0$ mm is $h \approx 2.25$ mm $\approx 0.9h_{\text{meas}}$. This yields a significant change in inductance, which is related to the overall scaling of the frequency change per change in gap distance $d$. Plotting experiment over the predictions in Figure 2.3, we can see the agreement between the lumped element model, simulation, and experimental implementation in Figure 2.6. By the use of a free parameter to determine the unflexed gap distance and allow for errors in cavity height $h$, the experimental data appears to follow a similar behavior as predicted in Figure 2.3.

The agreement of the theoretical and simulated predictions with my experiment gives a design protocol to follow for re-entrant cavity designs. In the mechanically compliant experiments, the gap distance $d$ is important to determine, and this model allows us to do so accurately. In addition, we
Figure 2.5: Squared Magnitude of transmission spectrum $|S_{21}|^2$ for the re-entrant cavity with different gap distances. The legend indicates the distance the adjustor screw displaced in the diaphragm.
Figure 2.6: Comparison of Experiment and Predictions of a re-entrant cavity resonance as a function of gap distance $d$. 
can estimate the frequency shift of the cavity given a change in $d$, which is essential to determining the electromechanical coupling, using the Fujisawa model. In the following chapters, I focus on loss mechanisms in re-entrant cavities, which is the primary focus of my thesis.
Chapter 3

Non-Radiative Power Loss in re-entrant Cavities

3.1 Loss Rate and Quality Factor

The figure of merit for underdamped oscillators can be either of two related quantities: the full linewidth at half-maximum $\kappa_{tot}$ and the quality factor $Q_{tot}$. The two quantities are related by

$$Q_{tot} = \frac{\omega_0}{\kappa_{tot}}$$  \hspace{1cm} (3.1.1)

where $\omega_0$ is the cavity resonant frequency. In the time-domain, $2\pi/\kappa_{tot}$ is the time required for the squared field amplitude to decay by a factor of $e^{-1}$. Therefore $\kappa_{tot}$ represents the energy dissipation rate in the cavity. Naturally, energy is dissipated through various mechanisms. The total loss rate is a sum of the individual loss rates:

$$\kappa_{tot} = \kappa_{ext} + \kappa_{int}$$  \hspace{1cm} (3.1.2)

Here, $\kappa_{ext}$ is the coupling rate to the measurement probe, which is either a magnetic dipole antenna or an electric dipole antenna fabricated by simply removing the material surrounding a short length of the center conductor of a UT-85 coaxial cable. The coupling rate is tunable by adjusting how much of the antenna is in the cavity. Note that coupling is still realizable even with the antenna recessed from the cavity entrance. The internal loss rate reflects energy dissipated due to uncontrolled degrees of freedom in the problem. This is written as:

$$\kappa_{int} = \kappa_c + \kappa_d + \kappa_r$$  \hspace{1cm} (3.1.3)
In words, the internal linewidth of a re-entrant cavity is the sum of the conducting linewidth, the dielectric linewidth, and the radiative linewidth, respectively. In this thesis I alternatively denote the internal linewidth as $\kappa \equiv \kappa_{int}$. Written in terms of quality factors, equations 3.1.2 and 3.1.3 are

$$\frac{1}{Q_{tot}} = \frac{1}{Q_{ext}} + \frac{1}{Q_{int}} \quad (3.1.4)$$
$$\frac{1}{Q_{int}} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r} \quad (3.1.5)$$

Similar to the loss rate notation, I denote the internal quality factor $Q_0 \equiv Q_{int}$.

The primary objective of this project in the Lehnert lab is to learn how to engineer an electrical cavity with an environmental coupling rate $\kappa_{int}$ that is smaller than the electromechanical single-photon coupling rate $g_0$. See Figure 1.1 for a visual representation of loss rates. In the following sections, I will briefly describe the physical mechanisms underlying conducting loss and dielectric loss for both ambient and cryogenic temperatures.

### 3.2 Conductor Losses

Because the materials we fabricate re-entrant cavities from have finite conductivity, the electric fields in the cavity will penetrate the metallic boundaries and dissipate energy in the conducting bulk. The extent to which an electromagnetic wave penetrates a conductor is called the skin depth. A larger skin depth results in a larger fraction of the field distribution that is dissipated via heat in the conducting bulk. Thus, the fraction of power dissipated in the conducting medium is proportional to the fraction of conducting volume the fields reside in to the total volume that the fields occupy. That is, $Q_c \propto V_{tot}/V_c$. The fraction of the conducting volume can be found by calculating the skin depth of the conductor and the surface area of the cavity. The skin depth can be approximated as $\delta = \sqrt{\frac{2\rho}{\omega \mu_0}}$, where $\rho$ is the resistivity of the metal and $\omega$ is the angular frequency of the AC current. For aluminum in the low GHz range, the skin depth is approximately one micron.

It is useful to compare this approximation with simulations and measurements. Using the cavity from Figure 2.2 as an example, $V_{tot}/V_c = 760$, whereas HFSS predictions suggest a $Q_c = 1600$. 
The measured unloaded $Q$ for this cavity was $Q_0 = 680$. The estimate that $Q_c \propto V_{tot}/V_c$ agrees within a factor of two of the simulation; this is sufficient as a consideration for new cavity designs.

To reduce conductor losses, we use materials that superconduct at low temperatures. The primary material we have used for re-entrant cavities is aluminum, alloy 6061, a type-I superconductor. The critical temperature $T_c$ is approximately 1.2 K.

3.3 Superconducting Losses

Superconducting materials offer a means of significantly reducing conductor losses, but superconducting materials still dissipate energy. The energy dissipated in a superconducting bulk material at very low temperatures is a step-like function of frequency. That is, for $T \ll T_c$, $T_c$ being the materials so-called critical temperature, all photons below a certain energy will be reflected at the boundary of the superconductor, with no power absorbed in the bulk. Above this energy, photons will be absorbed in the material, so the incident field has transferred some of its energy to the superconductor. This energy threshold is known as the energy gap in superconductors. For any non-zero temperature there is still some dissipation, even from photons with energy less than the gap, as described by the Mattis-Bardeen two fluid model. In the two-fluid model, there is a dissipative fluid of quasiparticles, and a lossless superfluid of cooper-pairs. The quasiparticles have density $n_{qp}$ and the superfluid particles have density $n_s$. As a function of temperature, $n_{qp}$ increases smoothly from zero at zero temperature, up to the material’s electron density at $T = T_c$. In contrast, $n_s$ smoothly decreases from its value at zero temperature down to zero at $T = T_c$. At zero-frequency (DC) the superfluid will carry all of the current and there will be no dissipation. At any non-zero frequency, however, the superfluid has an inductive character associated with the change in kinetic energy required to accelerate and decelerate the superfluid [7]. Consequently, the AC currents are carried partially by the normal fluid component and by the superfluid component, much like the currents in a parallel LR-circuit (see 3.3).

Another source of energy loss in superconducting physics is the presence of strong magnetic fields in the material before it is cooled below the critical temperature $T_c$. For both type-I and type-
The loss in the superconductor is zero at DC, which is analogous to a short-circuit by the inductor. At non-zero frequencies, current passes through the resistor. When \( T = T_c \), superconductivity is broken.

II superconductors, magnetic fields can become “trapped” in the superconductor as the temperature crosses \( T_c \). This trapped magnetic flux can move in the bulk of the superconductor due to the applied currents; when the trapped flux moves, there is a voltage induced by \( \varepsilon = -\frac{d\Phi}{dt} \), and so power is dissipated as \( P = I\varepsilon \). To provide a sense of scale of how many flux quanta could be present in a re-entrant cavity, consider the following calculation. The surface area of a re-entrant cavity is \( \sim 2 \times 10^{-4} \text{m}^2 \), and the Earth’s magnetic field is \( B_{Earth} \approx 0.25 \text{ G} \), so the total flux through a cavity from the earth alone is \( \Phi_{cav} \sim 5 \times 10^{-5} \text{G} \cdot \text{m}^2 \). This corresponds to \( 2 \times 10^6 \) flux quanta that penetrate the cavity; since the time-derivative of flux is proportional to how many flux quanta are present in the system, we know that a cavity with no magnetic shielding will in principle be limited by magnetic losses. Based on reports of magnetic losses giving \( Q_m \sim 1.5 \times 10^6 \) or more [11] in niobium re-entrant cavities, losses from trapped magnetic flux are not suspected as the current limiting loss mechanism of our re-entrant cavities.

### 3.4 Dielectric Loss

Aside from energy dissipation due to the materials we construct the cavity from, energy is also dissipated in materials that form when the cavity is exposed to atmosphere: dielectric oxide films. Energy dissipation in a dielectric material is characterized with two phenomenological descriptions: dielectric relaxation and two-level-system behavior in the material. Dielectric relaxation means
that the polarization of a molecule in the oxide lags in time with the applied electric field. The
time-constant associated with molecular polarization responses leads to dissipation in the dielectric
[6].

Dielectric loss in thin films has also been explained by the concept of saturable absorption
of two-level systems. The presence of adjacent asymmetric potential wells for atoms in an amorph-
ous (non-crystalline) material, leads to a two-level system formed by the asymmetric double-well
potential [14]. When a two-level system is driven very strongly, the two-level systems do not have
sufficient time to decay back to the ground state before being excited again. This makes the power
absorbed by these two-level systems saturate, as they cannot be excited to a higher energy. This is
in contrast to a harmonic oscillator, which has an infinite number of transitions at the same energy.
This two-level system behavior can account for the saturable nature of the dielectric loss observed
in amorphous dielectric. This characteristic of dielectric loss enables one to test whether dielectric
loss is dominanating the internal cavity loss rate by measuring $Q_0$ as a function of input power
amplitude.

In practice, the microscopic physics underlying dielectric power dissipation is not necessary
to calculate $Q_d$. For small gap distance $d$, $Q_d$ is well approximated by the loss between the center
post and upper cavity wall. The $Q$ of this capacitor is

$$Q_d = \frac{1}{\frac{d_{\text{mat}}}{d_{\text{vac}}} \tan \delta} \quad (3.4.1)$$

where $d_{\text{mat}}$ is the thickness of the amorphous oxide layer on the capacitor, $d_{\text{vac}}$ is the remainder of
the gap, occupied by vacuum, and $\tan \delta$ is the loss tangent of the thin film layer. The oxide layer
that forms on metallic aluminum in atmosphere is $\sim 4$ nm of alumina [1], whose loss tangent is
$\sim 10^{-3}$ at microwave frequencies. For an untreated aluminum surface at a gap distance of 100 nm,
for instance, $Q_d \sim 2.5 \times 10^4$. Though surface smoothness has been shown to minimally influence
the internal cavity linewidth in Niobium resonators , the root-mean-square surface variation of the
center post is critical to achieving center post gaps of 100nm or less. Further investigations of the
re-entrant cavity as a viable circuit with which to achieve strong electromechanical coupling would
certainly involve procedures to minimize surface roughness. This is critical to maximizing $g_0$. 
Chapter 4

Theory of Radiation Loss in Re-entrant Cavities

4.1 Background

In addition to power loss via absorption in the conductor and in dielectric films that form at the conductor-atmosphere interface, energy can radiate from the cavity. This would happen if there exists a waveguide that extends out to the environment and the fields in the re-entrant cavity can excite it.

For electromechanical cavities, the top consists of a metalized silicon wafer with a silicon nitride suspended membrane embedded in it. The membrane has square dimensions $\sim 500 \, \mu\text{m}$ and is mechanically compliant. Because we seek to fabricate the mechanical and electrical elements separately in the electromechanical setup, the cavities I assemble are assembled in largely the same way; one part contains the cavity volume and the excitation ports, while the other is the top. See Figure 4.1 for an example of a re-entrant cavity implemented in an electromechanical system. The distance between the mechanical membrane and the cavity is tuned using a JPE Cryo Actuator ”PiezoKnob”. Because the upper metal plate and cavity are fabricated separately and gently pressed together, they will not be connected by a perfect short circuit. This interface between the cavity and the membrane forms the parallel-plate transmission line. Though I studied a simpler geometry, the problem of radiation loss is still present because of the oxide film that forms on the cavity and upper metal plate. Because of the oxide film, current that flows up the outer cavity wall and reaches the interface is met with a non-zero impedance between the cavity lid and the pocket, and therefore a potential difference is formed at the junction of the two pieces. The
Figure 4.1: (a) Assembled Electromechanical system with a re-entrant cavity with the "Piezo-Knob" pictured. (b) Cavity with the Norcada silicon nitride membrane pictured. The suspended mechanical membrane is shown in the wafer center.
potential difference between two parallel-plates accommodates Transverse Electric and Magnetic (TEM) field modes, creating a radial parallel-plate transmission line. Figure 4.2 depicts how a potential difference between the cavity and the lid will propagate power out of the cavity. Re-entrant Cavities have been used to transmit the mechanical motion of a gravitational radiation antenna in attempt to measure gravitational waves [9, 11, 19]. These experiments required a gap between the re-entrant cavity and the mechanical oscillator, so the propagation of electromagnetic fields through gaps of this type has been studied before. I will begin with a transmission-line description of the parallel-plate radial waveguide before I address strategies to reduce radiation loss.

### 4.2 Cavity Interface Transmission-Line Analysis

We are interested in quantifying the energy dissipated through this radial transmission-line. Knowing the impedance of the line, we can calculate $Q_r$ in the same way as a series LRC circuit; the resistance in the problem, called the effective radiation resistance [11], is the real part of the impedance of the line seen from inside the re-entrant cavity. Once we know $R = \text{Re}[Z_{in}]$, we can calculate $\kappa_r$ as

$$\kappa_r = \frac{R}{L} \tag{4.2.1}$$

where $L$ is the inductance of the cavity, calculated using Fujisawa’s lumped-element models of re-entrant cavity geometries. Thus, $Q_r$ is calculated as

$$Q_r = \frac{\omega_0 L}{R} \tag{4.2.2}$$

Figure 4.3 shows the transmission-line model of the cavity interface. The terminated load $Z_L$ is the impedance of a parallel-plate transmission line radiating into free space, $Z_{in}$ is the impedance of the transmission line seen from the outer cavity wall, and $Z_0$ is the characteristic impedance of the transmission line. The input impedance of any lossless transmission line, derived from the telegrapher’s equations (see Pozer [15]), is given by:

$$Z_{in} = Z_0 \frac{Z_L + iZ_0 \tan(k_0 \Delta r)}{Z_0 + iZ_L \tan(k_0 \Delta r)} \tag{4.2.3}$$
Figure 4.2: A re-entrant cavity with a dielectric oxide formed on the conductor surface (dotted region). Therefore, a potential difference $V'$ between the cavity lid and bottom is formed. The interface of the lid and bottom forms a radial transmission line, with the electric field pointing in the (vertical) $\hat{z}$-direction. The waves propagate from the cavity to the environment, shown by the arrow.

Figure 4.3: Transmission-line picture of the radial waveguide surrounding the cavity. This supports TEM modes that propagate from the cavity to the surrounding environment.
In Equation 4.2.3 above, $\Delta r$ is the length of transmission line and $k_0$ is the wavenumber of the propagating wave. To determine the input impedance, we must first know the characteristic impedance of the transmission line and also the load-impedance of free space that terminates the line.

I will start with the impedance of the radial transmission line. The wave solutions for the boundary value problem can be found using a separation-of-variables approach to solving the Helmholtz equation in cylindrical coordinates. I will merely state the solution here. The traveling-wave solutions in cylindrical coordinates are Hankel functions, and the electric and magnetic fields are described as superpositions of inwardly and outwardly propagating forms of these special functions. Just as sines and cosines are expressed as an amplitude and phase of a complex exponential, Hankel functions can be expressed in an analogous way. The electric and magnetic field for the TEM mode of this transmission-line given below, are expressed using the amplitudes ($G_0(kr)$ and $G_1(kr)$) and phases ($\theta(kr)$ and $\psi(kr)$) of traveling-wave Bessel functions [17]:

$$E_z = G_0(kr) \left( Ae^{j\theta(kr)} + Be^{-j\theta(kr)} \right)$$

$$H_\phi = \frac{1}{\eta Z_0(kr)} G_0(kr) \left( Ae^{j\psi(kr)} - Be^{-j\psi(kr)} \right)$$

Here, $Z_0(kr) = \eta G_0(kr)/G_1(kr)$ is the wave impedance of the line and $\eta = \sqrt{\frac{\mu}{\epsilon}}$ is the impedance of the medium. It is intuitive that the electric field would point in the $\hat{z}$-direction, from one plate to the other. In addition, the surface currents that travel radially would generate magnetic fields in the $\hat{\phi}$-direction. The coefficients $A$ and $B$ can be determined knowing two field values at two different radii, though only their relative magnitudes influence the impedance of the line. Because we are driving the transmission line by the cavity within, I assume only outwardly propagating waves. Thus, $A = 0$ and $B$ cancels in determining $Z_r$ in 4.2.6. In terms of Bessel functions, $G_0(kr)$,
\( \theta(kr), G_1(kr), \) and \( \psi(kr) \) are given as:

\[
G_0(\nu) = \sqrt{J_0^2(\nu) + N_0^2(\nu)}
\]
\[
G_1(\nu) = \sqrt{J_1^2(\nu) + N_1^2(\nu)}
\]
\[
\theta(\nu) = \arctan \left( \frac{N_0(\nu)}{J_0(\nu)} \right)
\]
\[
\psi(\nu) = \arctan \left( \frac{J_1(\nu)}{-N_1(\nu)} \right)
\]

To find the characteristic impedance of the radial transmission line, we simply divide the voltage \( V \) by the current \( I \):

\[
Z_r = \frac{V}{I} = -\frac{b}{2\pi r} \left( \frac{E_z}{H_\phi} \right)
\] (4.2.6)

The impedance of this transmission-line depends inversely on the distance from its central axis \( r \) and linearly on the height \( b \) of the line. The admittance of a parallel-plate waveguide radiating into free space is given in [12] as:

\[
Y_{fs} = \frac{1}{Z_r} \left( \frac{k_0 b}{4} + i \frac{k_0}{2\pi} b \ln \left( \frac{4\pi e}{1.781k_0 b} \right) \right)
\] (4.2.7)

Taking the real part of the impedance \( \text{Re}[Z_{fs}] = \text{Re} \left[ Y_{fs}^{-1} \right] \) gives the radiation resistance:

\[
R_{rad} = Z_r \frac{k_0 b / 4}{(k_0 b / 4)^2 + \left( \frac{k_0 b}{2\pi} \ln(\frac{4\pi e}{1.78k_0 b}) \right)^2}
\] (4.2.8)

By substituting \( Z_r \) in 4.2.6 and \( R_{rad} \) in equation 4.2.8 for \( Z_0 \) and \( Z_L \) respectively in equation 4.2.3, we can estimate [11] an upper bound on the power radiated through the radial transmission-line, and therefore we can predict \( Q_r \). For the re-entrant cavity that I measured, the transmission line thickness was \( b = 50 \mu m \), the average characteristic radial waveguide impedance is \( Z_r \sim 0.15 \Omega \), and \( R_{rad} \sim 2 \Omega \). This gives an estimated \( Q_r \) of 600. Note that the length of the radial transmission line \( (r_5 - r_4) \) influences \( Q_r \). See Figure 4.4. For example, if the radial transmission line length were a quarter-wavelength, \( Q_r \sim 6000 \), while a transmission line that is a half-wavelength long will give \( Q_r \sim 30 \). For a re-entrant cavity with \( f_0 \approx 8 \) GHz, a \( Q_r \) of 6000 would correspond to a loss rate \( \kappa_r/2\pi \approx 1.3 \) MHz. This theory clearly suggests that a method to reduce radiation losses is needed should we seek \( g_0 > \kappa \).
Figure 4.4: $Q_r$ as a function of transmission line length in units of wavelength. The nominal values in used to calculate $Z_r$ and $R_{rad}$ are $b = 50$ µm, $f_0 = 7.86$ GHz, and the cavity dimensions specified in Figure 5.1.
4.3 Reducing Loss: The Choke Flange

The results of the theory suggests that radiation loss can limit $\kappa/2\pi$ to the order of MHz if it is not addressed. In order to reduce the power transmitted through the transmission line into the load impedance, one would want to reduce the coupling of the cavity to the transmission line. Since this coupling is caused by the voltage that forms across the cavity interface, minimizing this voltage would reduce the cavity coupling to the transmission line. Alternatively, one can think of this as forcing a current anti-node and a voltage node across the interface, meaning the current would see an effective short at the interface.

The issue of undesired radiative processes at conducting junctions is not new. Microwave engineers addressed the issue of radiation propagation at the interfaces formed by waveguide junctions. Their problem was that current flowing on the surface of a rectangular waveguide would build a potential difference at waveguide junctions, exciting the radial transmission line that was formed at the junction. This problem was addressed by installing a *choke flange* at the junction. A choke flange is an annular groove that is bored into one part of the interface. See Figure 4.5 for an example image of a choke flange.

The choke flange functions as a half-wave short-circuited transmission-line [10]. The choke groove (part (b) in Figure 4.5) is a quarter-wavelength in depth, which creates a large (ideally infinite) impedance at the groove entrance. Then, the second quarter-wave segment that goes from the groove entrance to the cavity entrance transforms the large impedance to a small one, thus reducing the potential difference across the interface. See Figure 4.6 for a diagram of the choke flange functionality. Effectively, the choke flange diminishes the coupling of the cavity to the radial transmission-line by attenuating the amplitude of the “voltage source” (see Figure 4.3) across the plates that drives the transmission-line. I will now develop the theoretical description of the choke flange.

The cavity geometry that will be considered in the next calculations is shown in Figure 4.7. The post is chosen to be conical because it is more feasible to fabricate as the top post-radius
Figure 4.5: Waveguide junction interface with a choke flange. Region (a) is the microwave rectangular waveguide. Region (b) is the flange groove that is bored into the interface. Region (c) is a gasket that allows the waveguide to be pressurized, if desired.
Figure 4.6: A re-entrant cavity cross section with a choke flange installed on the bottom part of the cavity. The coaxial groove forms a shorted quarter-wavelength transmission line, forming a large impedance at the groove entrance at the interface. A second quarter-wavelength transformer between the groove and the cavity reduces the voltage drop across the plates to almost zero. If tuned properly, this effectively decouples the cavity resonant mode from the radial transmission-line at the outer wall of the cavity.
$r_0$ becomes smaller. The addition of the choke flange changes the transmission-line diagram from Figure 4.3 to one in Figure 4.8. If the load impedance of the radial transmission-line in $r_2 < r < r_3$ is very high, then there will be a current node at $r_3$ and hence a voltage node at $r_2$. This would maximize current across the interface at $r_2$, reducing radiation loss substantially.

We can calculate $Z_{in}$ from equation 4.2.3, using $Z_L = Z_g + R_{Rad}$, where $Z_g$ is the impedance of the groove. The groove is a shorted coaxial transmission line, where the inner conductor radius is $r_3$ and the outer conductor radius is $r_4$. Thus, $Z_g$ is the input impedance of a shorted quarter-wave coaxial line, and can be calculated using the equation 4.2.3 where $Z_L = 0\Omega$:

$$Z_g = iZ_d \tan (k_0 d)$$  \hspace{1cm} (4.3.1)

where $k_0$ is the wavenumber of the cavity resonance, $d$ is the depth of the groove, and $Z_d$ is the coaxial line characteristic impedance:

$$Z_d = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln \left(\frac{r_4}{r_3}\right)$$  \hspace{1cm} (4.3.2)

The radiation resistance $R_{rad}$ is equation 4.2.8 where $Z_r$ is the characteristic impedance of the radial transmission line in the region $r_4 < r < r_5$. Thus, the load impedance terminating the radial transmission line is

$$Z_L = iZ_d \tan (k_0 d) + R_{rad}$$  \hspace{1cm} (4.3.3)

So, the input impedance into the radial line with a choke flange is

$$Z_{in} = Z_r \frac{(iZ_d \tan (k_0 d) + R_{rad}) + iZ_r \tan (k_0 (r_3 - r_2))}{Z_r + i(iZ_d \tan (k_0 d) + R_{rad}) \tan (k_0 (r_3 - r_2))}$$  \hspace{1cm} (4.3.4)

As before, $Q_r$ is $\omega_0 L / Re[Z_{in}]$. In Figure 4.9, I plot $Q_r$ as a function of the choke flange resonance ($f_{CF} \approx c/4d$, where $c$ is the speed of light) normalized by the cavity resonance. As expected, when there is perfect tuning between the choke flange and the cavity resonances, $Q_r$ diverges. It is desirable to compare theoretical sensitivity to cavity-choke detunings with measurements, as this will define tolerances to which a choke flange resonance must align with the cavity resonance at the desired center-post gap distance.
Figure 4.7: Re-entrant Cavity Geometry with a radiation choke flange embedded. The dimensions shown are referenced in the text describing radiation losses in the presence of an RF choke. The dashed lines show the coupling ports for microwave probes into the cavity.

Figure 4.8: Transmission-line picture of the interface between the cavity pocket and the cavity lid with a choke flange embedded in one interface. The radial transmission line length is \((r_3 - r_2)\), and is terminated with the groove impedance \(Z_g\) in series with the radiation resistance \(R_{rad}\).
Figure 4.9: Theoretical plot of $Q_r$ as a function of the choke resonance $f_{CF} \approx c/4d$, normalized by the cavity resonance. The values used in equation 4.2.3 are the same as in Figure 4.4 and the coaxial impedance $Z_d \approx 15 \Omega$. 
In the following chapter, I will present my experimental investigation of $Q_r$ in re-entrant cavities. Ultimately, I seek to identify whether radiation is a dominant source of power loss for our electromechanical systems. In addition to this primary goal, my experimental observations will comment on the effectiveness of the choke flange at different choke flange frequencies.
Chapter 5

Measuring Radiated Power

My goal is to determine $Q_r$ for re-entrant cavities, and to investigate how choke flange designs can potentially improve $Q_r$. Thus, I will determine $Q_r$ without a choke flange and with a choke flange. Because the choke groove and radial transmission line forms a *half-wave* impedance transformer, the choke flange performance is a function of frequency. The detuning between choke and the cavity thus impacts the choke’s performance. Therefore, it is of interest to measure $Q_r$ for various cavity-choke frequency detunings. Below, I describe the procedure to determine $Q_r$. Then, I will present my findings which evaluate $Q_r$ as the relative frequency tuning between the cavity and the choke flange is varied.

5.1 Approach

To determine whether or not radiation is a dominant loss mechanism in superconducting re-entrant cavities, I need a way to measure power that the cavity radiates. The natural measurement tool here is an antenna. The antenna can be seen as a second “port”, that forms a two-port microwave network. Measuring the forward transmission coefficient through this network would inform me how strongly coupled the cavity is to the antenna; that is, a large transmission coefficient near unity indicates a substantial amount of power is radiating from the cavity. However, antennae are finite in size and have imperfect efficiency; therefore, the power I pick up in the antenna $P_a$ is only a fraction of the total power radiated $P_r$. So,
\[ P_a = \gamma P_r \] (5.1.1)

where \( \gamma \in [0, 1] \) is the fraction of the total radiated power that the antenna measures. In general, \( \gamma \) depends on the radiation pattern and efficiency of both the receiving antenna and the cavity that constitutes the transmitting antenna. In section 5.3, I address how I determine \( \gamma \) experimentally.

The radiative \( Q \) and the radiated power are related simply by the definition of the quality factor:

\[ \frac{Q_r}{Q_0} = \frac{P_c}{P_r} \] (5.1.2)

where \( P_c \) is the power that enters the cavity through the excitation probe. The power that enters the cavity can be determined knowing the incident and reflected power. That is,

\[
\left| \frac{P_c}{P_{in}} \right| = 1 - \left| \frac{P_{ref}}{P_{in}} \right|
= 1 - |S_{11}|^2
\] (5.1.3)

Thus, we obtain an expression to measure \( Q_r \). All we need to know is the unloaded quality factor \( Q_0 \), the reflection coefficient on resonance, the forward transmission coefficient on resonance, and the antennae coupling efficiency \( \gamma \). By combining and rearranging equations 5.1.2 and 5.1.3, we have:

\[ Q_r = \gamma \frac{1 - P_{ref}}{P_a} Q_0 \] (5.1.4)

In terms of reflection and transmission coefficients, equation 5.1.4 is

\[ Q_r = \gamma \frac{1 - |S_{11}|^2}{|S_{21}|^2} Q_0 \] (5.1.5)
5.2 Measurement Setup

An important design choice in this experiment is what type of antenna to measure radiation with. One that is commonly used to measure the characteristics of other antennae is an electromagnetic horn. The horn antenna consists of a rectangular waveguide and a tapered section between free space and the waveguide. The antenna can be coupled to a microwave measurement device, such as a vector network analyzer, using an electric or magnetic dipole antenna that perturbs the waveguide. The tapered section, which gives the horn its name (see Figure 5.1 for an example horn antenna), matches the impedance between the waveguide and free space by gradually changing the waveguide dimensions. This reduces wave reflections that occur at the waveguide boundary and, by extent, the power these reflections would dissipate in the waveguide due such resonances. Thus, the horn transmits power from free space to the waveguide and vice versa with low loss. Horn antennae are also highly directional; only signals that have components directed in the horn’s mouth will be transmitted. This means that using a horn rather than a low-directivity antenna (such as a dipole) offers a higher signal-to-noise ratio.

In the far-far-field measurement scheme, the high directivity of the horn would result in high sensitivity to misalignments between the transmitting antennae and the horn, resulting in larger
systematic errors. However, when the antenna is very near the cavity, such as in Figure 5.3, the aperture is a large fraction of the solid angle seen from the cavity center. This makes the horn insensitive to misalignments and variations in the cavity radiation pattern. Though there was concern that coupling the measurement antenna could in fact lower the cavity $Q_0$ by creating a lower impedance than free space at the cavity boundary, no change was measured in the $Q_0$ when the antenna was placed nearby and when it was not.

A schematic and microwave network diagram of the measurement setup is given in Figure 5.2. The horn is the receiving port of the two-port microwave measurement, with the coupling antenna in the cavity serving as the transmitting port. To isolate the measurement from conductors and other materials that would alter the radiation pattern, the cavity is elevated on an acrylic pedestal. To align the horn, it is clamped to a post with adjustable height. Figure 5.3 depicts the measurement setup. To maximize the coupling between the horn and the cavity, the mouth of the horn is brought tangent to the cavity, with the E-plane of the horn aligned with the electric field mode of the radial transmission line. The horn antenna was a Pasternack WR-137 horn with 10 dB of gain, with the bandwidth ranging from 5.85 GHz to 8.2 GHz. Antenna gain is merely the product of its efficiency and its directivity, where the latter expresses how directional an antenna is, and the former is a measure of how lossy the antenna is. The horn’s beam-width was 55.1° in the E-plane (vertical) and 54.2° in the H-plane (horizontal). It should be noted that only large misalignments ($> 50°$) between the horn mouth and the cavity saw noticeable signal reduction. Also, rotating the horn such that the E-plane was perpendicular to the transmission line electric field mode resulted in no signal above the noise floor ($\sim -80$ dB). The reflection and transmission coefficients were measured using a vector network analyzer. The measurement lines were calibrated up to the Miteq microwave signal amplifier (38 dB of measured gain) coming out of the horn and the coaxial feed line entering the cavity.
Figure 5.2: (a) Two-port microwave network. The S-parameters, labeled in the interior as $S_{ba}$, are the ratios of $V_{out}$ of port $b$ to $V_{in}$ of port $a$. (b) Measurement schematic of finding $P_a$. The black box surrounding the schematic makes an analogy to (a): the horn is the second port to the microwave network, with $P_a$ being equivalent to $P_{out}$ in (a).
Figure 5.3: Measurement Setup for Calculating $Q_r$. For measurements of $Q_r$ with a choke flange, I put a Miteq amplifier (gold box) at the horn’s output.
5.3 Experiment

Using the configuration detailed in the previous section, I measured $Q_r$ for a re-entrant cavity that was assembled in several ways. First, the cavity with no choke flange was measured, and then the cavity was measured with one of several choke flanges surrounding it. Each flange was tuned to a different nominal frequency. To do this, the choke flanges were installed into the upper wall, rather than into the cavity as depicted previously. See 5.4 for an image of the cavity that was measured. The cavity interface was mechanically polished with sandpaper of increasing grit up to 1000, followed by polishing the cavity pocket using Semi-Chrome polish. Then, the cavity was cleaned in an ultrasonic bath, rinsed with deionized water, dried with compressed air, rinsed with methanol, and dried with compressed air once more.

![Figure 5.4: Photo of re-entrant cavity used to measure radiation loss. The cavity post is conical, rather than cylindrical, due to practical considerations in machining a small post area. The dimensions listed correspond to those defined in Figure 4.7. The cavity capacitance $C$ and inductance $L$ were calculated using the lumped-element model for conical re-entrant cavities \[4\].](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$</td>
<td>0.02 mm</td>
</tr>
<tr>
<td>$r_0$</td>
<td>0.465 mm</td>
</tr>
<tr>
<td>$r_1$</td>
<td>2.59 mm</td>
</tr>
<tr>
<td>$r_2$</td>
<td>8.67 mm</td>
</tr>
<tr>
<td>$r_5$</td>
<td>25.4 mm</td>
</tr>
<tr>
<td>$h$</td>
<td>4.06 mm</td>
</tr>
<tr>
<td>$L$</td>
<td>1.5 nH</td>
</tr>
<tr>
<td>$C$</td>
<td>0.43 pF</td>
</tr>
</tbody>
</table>

The cavity was fastened together using Kapton tape around the rim (see Figure 5.3); threaded metal fasteners were not used because they substantially alter the geometry of the transmission line, possibly changing the mode structure. The surrounding Kapton tape is shown to have negligible
effect on the radiation pattern. To be sure of the transmission line thickness $b$ and to increase signal strength, a ring of PTFE with thickness $\sim 50 \mu m$ was placed along the outer rim of the cavity interface. The ring was approximately $1.5 \text{ mm}$ wide. The dimensions of the choke flanges are given in Table 5.1.

The cavity is outfitted with two probe port holes entering the bottom, though the cavity was excited in reflection. During measurement, the second port hole is left empty. Radiation out of this probe hole is not a concern because the largest wavelength allowed in a circular waveguide is $\lambda = \pi d/p_{11}^\prime$, where $p_{11}^\prime$ is the first zero of $J_n^\prime(z)$, the derivative of the bessel function of the first kind [17]. Therefore, the largest wavelength that can propagate in the probe hole. The hole diameter is $d \sim 4.4 \text{ mm}$, which corresponds to a minimum frequency of $f \sim 81 \text{ GHz}$.

To estimate $\gamma$, I assume $Q_0$ is dominated by $Q_c$ and $Q_r$ for a cavity with no choke flange. This is reasonable because the large gap spacing diminished $Q_0$ from $\sim 700$ with no gap spacing to $\sim 250$ with the gap spacing, indicating more power is radiating from the cavity. In addition, room-temperature cavities have never exceeded the HFSS simulation prediction of $Q_c \sim 3000$ for the geometry presented in Figure 5.4, and the participation ratio of lossy dielectric to vacuum in the cavity is small enough such that, using equation 3.4.1, $Q_d \sim 10^7$ at center-post gaps $x_0 \approx 25 \mu m$.

Assuming $Q_c \approx 3000$, as HFSS predicted for the cavity geometry in 5.4, I calculate $Q_r$ using the estimate

$$Q_r^{-1} \approx Q_0^{-1} - Q_c^{-1}$$

(5.3.1)

I fit the cavity reflection squared response $|S_{21}|^2$ to the Lorentzian fit function in equation 5.3.2:

$$|S_{11}|^2 = \frac{(\kappa_{int} - \kappa_{ext})^2 + 4 (\omega - \omega_0)^2}{(\kappa_{int} + \kappa_{ext})^2 + 4 (\omega - \omega_0)^2}$$

(5.3.2)

From the fit (see Figure 5.5), I estimated $Q_0 = \omega_0/\kappa_{int} \approx 230$ for the cavity without a choke flange. Using equation 5.3.1, I estimate $Q_r \approx 270$ without a choke flange. Furthermore, using the measurement scheme in Figure 5.2, I measure the transmission coefficient of the cavity through the horn on resonance. Using the results of these measurements in equation 5.1.5, I calculate
Table 5.1: Dimensions of each choke flange. The first column include pictures of each flange. Column 2 is the radial distance from the cavity outer wall to the inside radius of the choke flange. Column 3 is the width of the groove, and column 4 is the depth. Column 5 gives the characteristic impedance of the coaxial line.

<table>
<thead>
<tr>
<th>Choke Flange</th>
<th>$(r_3-r_2)$ [mm]</th>
<th>width [mm]</th>
<th>Depth [mm]</th>
<th>$Z_d$ [Ω]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12.66</td>
<td>2.58</td>
<td>13.41</td>
<td>6.81</td>
</tr>
<tr>
<td></td>
<td>11.36</td>
<td>2.87</td>
<td>11.49</td>
<td>7.98</td>
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<td></td>
<td>9.63</td>
<td>5</td>
<td>9.88</td>
<td>14.4</td>
</tr>
<tr>
<td></td>
<td>7.25</td>
<td>4.64</td>
<td>7.03</td>
<td>15.24</td>
</tr>
</tbody>
</table>
Figure 5.5: (a) Reflection coefficient squared magnitude $|S_{11}|^2$ for a cavity with no choke flange and a 50 µm gap. (b) The transmission coefficient squared magnitude $|S_{21}|^2$ through the horn antenna for the same cavity as (a). Because $Q_r$ and $Q_c$ dominate $Q_0$, this measurement provided an estimate for $\gamma$. 
\( \gamma \approx 2 \times 10^{-2} \). Although \( \gamma \) could differ in the subsequent measurements due to alterations in the radiation pattern imposed by the choke flange, this would not change \( \gamma \) by an order of magnitude.

### 5.4 Results

With an estimate of the coupling \( \gamma \) between the cavity and the horn antenna, I can measure how \( Q_r \) varies with choke flanges tuned to different frequencies. In principle, I would like to have one choke flange and change only its groove depth \( d \). However, a change in the flange resonant frequency \( f_{CF} \) also requires adjusting the length of the radial transmission line between the groove and the cavity. Because the cavity outer diameter is fixed and I sought to measure the same cavity for consistency, there were constraints on the choke flange fabrication process. Because of the fixed cavity outer diameter \( r_5 \), I needed to make the groove depth thinner for a low-frequency choke to accommodate for the longer radial section. This reduced the coaxial impedance \( Z_d \) (see Table 5.1). Figure 5.6 shows the choke flanges that were used in the each cavity assembly.

The radiative quality factors for the choked configurations are presented in Figure 5.7. Because each choke flange has a different groove thickness, and therefore different coaxial characteristic impedance, a single theoretical curve of \( Q_r \) versus \( f_{CF}/f_0 \) to compare to measurements is not appropriate. These measurements clearly show the improvement in cavity \( Q_r \) when a choke flange is placed around it. If I conservatively suppose that I am actually picking up half of the power I estimated such that \( \gamma \sim 10^{-2} \), my measurements indicate it is possible to achieve \( Q_r \sim 10^7 \) with a 50 \( \mu \)m gap between the cavity and the upper wall. This gives a ratio of radiative to internal quality factors \( Q_r/Q_0 \sim 10^5 \). For the simple geometry that I studied, radiation will certainly not be the dominant source of loss when a properly-tuned choke flange is implemented in the system. In addition, it provides information about the frequency dependence of the choke effectiveness. An important feature to note is that the experimental data is maximum when the choke-cavity detuning \( f_{CF}/f_0 \sim 0.8 \). This result suggests that the resonant frequency of the choke flange is lower than expected. In other words, the length of coaxial transmission-line is slightly larger than the choke depth \( d \). This extra length can be accounted for in the width of the coaxial groove. The
Figure 5.6: A group photo of the cavity with each choke flange measured. Because of the constrained cavity diameter, fabricating low-frequency choke flanges required me to change the groove width and hence the coaxial impedance.
Figure 5.7: Radiative $Q$ vs choke-cavity detuning.
current travels across the groove bottom, and there is a potential difference along the bottom of the groove. So contrary to the theoretical description of the coaxial groove, the current must travel a distance greater than \( d \) to reach a short circuit (i.e. a voltage node). Supposing the coaxial groove depth \( d \approx d + \epsilon \), where \( \epsilon \sim 0.5(r_1 - r_3) \), it seems the discrepancy between the measured \( Q_r \) values and the theoretical ones can be explained. As shown in Table 5.1, the highest-frequency choke has the highest width:depth ratio; it is more than half as wide as it is deep. Thus, the actual resonant frequency of the choke could be up to 30% less. This would explain why the measured \( Q_r \) for this cavity is much larger than predicted.

The analysis described in the preceding paragraph does not seem to hold for the \( f_{CF} \sim 0.8f_0 \) choke flange. Adding a hypothetical length of coaxial transmission line to the two lower flanges introduces a larger discrepancy between the model and the data. It seems that more measurements should be made to investigate the choke flange resonance as a function of groove width, in addition to groove depth. In the next chapter, I will briefly discuss further experiments I feel would provide concrete understanding of radiation loss. These experiments would shed light on the upper bounds of \( Q_r \) when using a choke flange, and if one can eliminate radiation loss from re-entrant cavities.
Chapter 6

Conclusions and Outlook

The goal of my experiment was to estimate the scale $Q_r$ for a simple re-entrant cavity geometry. Supposing radiation would become the limiting factor after having addressed the other loss mechanisms in the cavity, determining a lower bound of $Q_r$ informs the conceivability of $g_0 > \kappa$. My experiment has shown the re-entrant cavity is capable of $Q_r \sim 10^7$ with a radiation choke flange in place. Supposing the internal quality factor $Q_0$ of the re-entrant cavity were limited by $Q_r \sim 10^7$, then, the cavity linedwidth $\kappa/2\pi \approx 800$ Hz for $\omega_c/2\pi \approx 8$ GHz. Using this as a guide to determining the necessary center-post gap distance for a cavity with $r_0 \sim 2.5$ $\mu$m, we would need a gap distance $x_0 \sim 2$ nm. This gap spacing is roughly two orders of magnitude smaller than the surface variation of the center post using the current fabrication scheme, though there do exist technologies capable of producing surfaces with this smoothness. To accommodate for realistic gap spacing between the suspended membrane and the cavity center post, one would seek an internal quality factor at least one order of magnitude higher, such that the gap spacing would be 20 nm.

How should one proceed? First and foremost, a measurement of $Q_r$ versus the detuning of cavity and choke frequency that is more thorough is necessary to find an upper bound on $Q_r$ for a given transmission line thickness $b$. In order to clearly verify the predictions presented in Figure 4.9, I would need a more dense data set. One proposal of how to obtain such a dataset is to vary the cavity resonance, rather than the choke flange resonance, as the cavity resonance is more naturally suited for tuning via a flexible diaphragm, much like that shown in 2.4. Indeed, this design has its own problems to solve, but it offers a high-resolution dataset that would outline in more detail
what the maximum $Q_r$ is for a cavity with a choke flange around it, and also it would more clearly resolve the discrepancy in frequency seen in Figure 5.7.

In addition to further investigation of the choke flange, the question of alternative approaches that can eliminate radiation loss altogether is raised. Aside from my investigation of $Q_r$ with a choke flange in between the cavity and the environment, which creates a current node at the interface near the cavity boundary, I have explored procedures to solder a cavity together, thus eliminating the transmission line completely. In particular, I plated a Lead-Tin (Pb/Sn) alloy onto copper re-entrant cavities, as Pb/Sn is a known superconductor ($T_c \sim 7$ K). A photograph of a copper re-entrant cavity I designed, as well as the Pb/Sn plated cavity are shown in Figure 6.1. How one can solder a mechanically-compliant wafer with a suspended membrane is one concern regarding this approach to eliminating radiation loss. Furthermore, exploring novel materials to fabricate re-entrant cavities is an important step toward reducing other losses, such as dielectric loss.

![Figure 6.1: (a) A re-entrant cavity made from oxygen-free high-conductivity (OFHC) copper. The visible scratches on the surface can be removed via electropolishing. (b) A Lead-Tin plated OFHC copper re-entrant cavity of the same design. By soldering together the cavity and lid, radiation loss can be prevented entirely.](image)

To achieve $g_0 > \kappa$, dielectric loss must be addressed. For small gap spacings, such as $x_0 \sim 100$ nm, the dielectric loss will dominate $\kappa$. This is the same problem that vacuum-gap capacitors face,
though the 3-D microwave cavity opens options for procedures to reduce the prevalence of thin lossy films that form on the conducting surface. Assuming this problem can be solved, further investigation about radiation loss is still necessary to be confident the scale at which radiation will limit the electrical quality factor. My measurements have shown that $Q_r \sim 10^7$ is achievable, though a more dense dataset that can determine the maximum $Q_r$, giving us more knowledge of the capabilities re-entrant cavities have as high-$Q$ electrical circuits.
Bibliography


